Telescope 51: What a Rule Can See

Recovery thresholds and finite witnesses on a fixed record

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Abstract. Recovering a target from an observation requires that the target be constant on every observation fiber. For a finite chain of refinements, the sufficient levels form a terminal interval. Its first level is the greatest separation depth of a target-distinct pair; one pair therefore witnesses the failure of every coarser level. We give exact witnesses on a fixed 51-symbol record: a swap hidden by card value and exposed by rank, equal resolution summaries with different labeled winners, and equal signed totals with different prefix paths. The component alphabet and its provenance belong to Formula 51; joint recovery and decisions belong to Project 51. Here the purpose is to isolate the threshold and make its hypotheses inspectable. The results concern declared maps, not zero prediction or a new interpretation of entropy.

1. State, observation, and target

Let $K : X \to Y$ be an observation and $Q : X \to Z$ a target. Write

$$\ker K = \{(x, x') : K(x) = K(x')\}.$$

Recovery means that $Q = h \circ K$ for a function h on the attained image $K(X)$. Thus

$$Q \text{ is recoverable from } K \quad \iff \quad \ker K \subseteq \ker Q. \tag{1}$$

Necessity follows by applying h to equal readings. For sufficiency, define h as the common target value on each fiber. This elementary factorization criterion is the starting point inherited from *The Line Game* [1]. It does not require probabilities or linear structure.

A finite family $K_j : X \to Y_j$, $j = 0, \ldots, m$, is a **refinement chain** when the attained observations satisfy $K_j = \pi_j \circ K_{j+1}$. Equivalently,

$$\ker K_m \subseteq \cdots \subseteq \ker K_0.$$

The maps may retain more information without changing the underlying state.

Theorem 1 (Recovery threshold). Suppose Q is recoverable from K_m. The levels that recover Q are exactly $\{j_*, \ldots, m\}$, where

$$j_* = \min\{j : \ker K_j \subseteq \ker Q\}. \tag{2}$$

If Q is nonconstant, define

$$d(x, x') = \min\{j : K_j(x) \neq K_j(x')\}, \qquad Q(x) \neq Q(x').$$

Then

$$\boxed{j_* = \max_{Q(x) \neq Q(x')} d(x, x').} \tag{3}$$

For a constant target, $j_* = 0$; no maximum over an empty set is needed.

Proof. Refinement can only shrink fibers, so an inclusion in (2), once true, stays true. Every target-distinct pair is separated at level m by the hypothesis. Once separated it stays separated, so its separating levels are precisely $d(x, x'), \ldots, m$. Level j recovers the target exactly when it has separated every such pair. This gives (3). Even if X is infinite, the nonempty set of depths lies in the finite set $\{0, \ldots, m\}$, so its maximum is attained. □

Corollary 1 (Persistent witness). If $j_* > 0$, some pair has equal observations at $j_* - 1$ and different targets. That same pair defeats every level $i < j_*$.

The obstruction is equality of readings, not insufficient reporting precision. The theorem gives a threshold along the declared chain. It does not select the best chain or bound the cost of finding the threshold.

2. A card realization, and a separate decision rule

Use the ordinary faces in reference order A,2,...,K, with suits ♠, ♡, ♢, ♣, and omit 2♣. The 51 remaining identities form the active alphabet; $I =$ 2♣ stays outside it. A symbol map acts coordinatewise on an ordered record.

Let V give A value 1, ranks 2–9 their face values, and 10,J,Q,K value 10. Let R be natural rank A=1 through K=13; C is signed color, black -1 and red $+1$; and S is suit identity. The raw maps V, R, C, S are not a refinement chain. The cumulative observations are:

$$K_0 = V, \qquad K_1 = R, \qquad K_2 = (R, C), \qquad K_3 = (R, S). \tag{4}$$

Rank determines V, and suit determines color. Here V is a fixed denomination map, not the contextual soft-ace value of a blackjack hand.

Telescope's **winner rule** is an additional convention. Divide a record into 17 consecutive triples, with positions labeled P1,P2,P3. Maximize value, then natural rank, then signed color, then suit order ♠, ♡, ♢, ♣; each later stage acts only on the leaders still tied. Formally,

$$L_{-1} = \{1, 2, 3\}, \qquad L_j = \underset{p \in L_{j-1}}{\arg\max}\, f_j(p). \tag{5}$$

Stop when one leader remains. This is lexicographic comparison. Refinement describes distinguishability; tie-only maximization describes winner selection. A finer score need not preserve an earlier winner unless this protocol is imposed.

The fixed record, with row breaks used only for reading, is

A♠ A♡ A♢ A♣ 2♠ 2♡ 2♢ 3♠ 3♡ 3♢ 3♣ 4♠ 4♡
4♢ 4♣ 5♠ 5♡ 5♢ 5♣ 6♠ 6♡ 6♢ 6♣ 7♠ 7♡ 7♢
7♣ 8♠ 8♡ 8♢ 8♣ 9♠ 9♡ 9♢ 9♣ Q♢ K♡ 10♡ K♢
Q♡ Q♠ J♠ 10♣ J♣ Q♣ K♣ 10♠ J♢ J♡ K♠ 10♢

Call this branch A. Branch B exchanges only positions 50 and 51, K♠ and 10♢. The zero-based Lehmer decoder assigns

$$\mathcal{R}(x) = \sum_{i=1}^{51} d_i (51 - i)!,$$

where d_i counts unused reference identities preceding x_i. It gives

$$\mathcal{R}(A) = 14\,134\,725\,141\,735, \qquad \mathcal{R}(B) = \mathcal{R}(A) - 1.$$

With scale 10^{12} these decode to the stored decimals 14.134725141735 and 14.134725141734. This is transport of a supplied approximation, not computation or certification of a zero.

3. Three witnesses

3.1 A pair separated by refinement

The two swapped identities obey

$$V(\mathrm{K}\spadesuit) = V(10\diamondsuit) = 10, \qquad R(\mathrm{K}\spadesuit) = 13 \neq 10 = R(10\diamondsuit).$$

Their separation depth in (4) is 1. For the target natural rank, they are a persistent witness against K_0. The full natural-rank target first becomes recoverable at K_1.

3.2 Equal summaries, different labeled outcomes

Let D record the first resolving stage of each triple, M the candidate counts after each stage, and W the labeled winner history. The supplied replay computes

$$D(A) = D(B) = \texttt{SCCV SSCV SSCV SCRRR}, \tag{6}$$

$$W(A) = \texttt{3333 2333 2333 31312}, \qquad W(B) = \texttt{3333 2333 2333 31313}. \tag{7}$$

Only the final triple changes:

$$A : (\mathrm{J}\heartsuit, \mathrm{K}\spadesuit, 10\diamondsuit), \qquad B : (\mathrm{J}\heartsuit, 10\diamondsuit, \mathrm{K}\spadesuit).$$

Rank selects the king at P2 in A and P3 in B. Both final candidate trajectories are $(3, 3, 1, 1, 1)$, including the initial count; all previous triples coincide. Hence $M(A) = M(B)$ while $W(A) \neq W(B)$.

Proposition 2. On any record domain containing A and B, neither D nor M, nor their joint observation (D, M), recovers W.

Proof. The two records share each stated summary and have different targets, contradicting (1). □

These are maps on whole records, not members of the symbol-level chain (4). The proposition applies the same fiber criterion at a different level. The repeated opening depth block also has a codec explanation: ranks below 16! fix the first 35 identities; eleven triples and the value resolution of the twelfth are forced. It is not evidence of periodic zero ordinates.

3.3 An endpoint hides a path difference

The active pack has 26 red and 25 black identities. Its signed sum is therefore $+1$ in every order; restoring the outside black marker gives zero. Let $S_n = \sum_{i=1}^{n} C(x_i)$, with $S_0 = 0$.

For an additive map into an abelian group, swapping adjacent entries at $k, k+1$ changes only prefix k, by $f(x_{k+1}) - f(x_k)$. Prefixes before it contain neither entry and prefixes after it contain both. Thus

$$(S_{49}, S_{50}, S_{51})_A = (1, 0, 1), \qquad (S_{49}, S_{50}, S_{51})_B = (1, 2, 1). \tag{8}$$

The endpoint agrees while the path differs. The full prefix path recovers the color word by successive differences. Its parity does not: $S_n \equiv n \pmod 2$ for every ordering. Project 51 develops the signed second-difference reading of this same witness [4].

4. What the alphabet and capacity supply

Formula 51 [3] fixes a uniform model on the first 1,438 Hangul syllables, decomposes them into components, and computes expected conditional information contributions. The resulting exact class sizes are

$$2, 1, 9, 1, 9, 11, 18.$$

The radix, encoding, probability model, and sort are declared conventions. The ordinate values do not enter the derivation. This paper uses that dictionary; it does not repeat its provenance analysis or claim that 51 was obtained by an optimization over games.

Permuting the final 18 identities while fixing the first 33 preserves the entire positional weight profile and gives 18! distinct records. Equivalently, these are the ranks $0 \leq \mathcal{R} < 18!$. All 1,566 original stored ordinate decks, at scale 10^{12}, lie there. Equal weight profiles do not make their ordered records equal.

There is also an independent **address-capacity** comparison. Using the archived LMFDB count $N = 103\,800\,788\,359$ quoted in v0.6 from the 20 September 2026 source snapshot,

$$11! < N < 15! < 18!.$$

Only the 18-member class among the seven has enough permutations to label every index individually. This says that an injection exists. Retrieving an ordinate still requires the index convention and the external archive. The archive's terminal stored ordinate at twelve decimal places does not itself fit into this 18! sector. No new audit of the archive's completeness is claimed here.

5. Scope and reproduction

The general threshold is an elementary consequence of factorization and nested fibers. Its value here is a precise test with witnesses that can be laid out by hand. The rule fixes which distinctions are observed; the target fixes which distinctions matter to recovery. Assigning payoffs or pricing a reveal introduces an additional decision problem, treated in Project 51 [4].

The accompanying package retains `T51_replay.py`, `derive51.py`, the 51-row table, and the exact card string. The replay regenerates (6)–(8), ranks, and candidate ledgers. The derivation checks weight equalities and the component-plus-END ledger exactly. Version 0.7 retains the v0.6 mathematics, shortens repeated method history, updates the reading links, and states the constant-target case and joint-summary obstruction explicitly.

References

1. J. L. Huckstead, *The Line Game / Surprise Is Not Meaning*, v7.5, corrected 24 September 2026, §§4–6. Zenodo record 22943070.
2. J. L. Huckstead, *Telescope 51*, v0.6 and supplied replay material, in Zenodo record 22970030. The record's deposit version and this manuscript's version are separate identifiers.
3. *Formula 51: Where the 51 Symbols and the 18-Class Come From*, corrected method record, 23 September 2026; concise review draft dated 26 September 2026 in this package. Original computational derivation credited to the Constructive Reasoning Seat.
4. J. L. Huckstead, *Project 51: Reading the Record Across Rules*, v0.4 review draft, this package; predecessor v0.3 in record 22970030.
5. LMFDB Collaboration, Completeness and Source of Riemann zeta-zero data. Archive figures above retain the source snapshot cited in v0.6; the cards do not extend it.

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