Project 51: Reading the Record Across Rules

Joint recovery, useful reveals, and Prospect 51

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Abstract. Project 51 studies what a declared view of an ordered record retains, and when a missing distinction changes a decision. On a fixed 51-identity alphabet, component-weight class and reduced polar address leave exactly eight compatible arrangements per complete positional profile. We retain that decoder and sharpen the sector result: polar address with signed color is injective throughout the 23! initial factorial sector; adding weight class extends this to 45!. Both bounds are sharp among these sectors. Prospect 51 then supplies a decision witness: two hands share their weight-and-polar observation but have different unique optimal holds. Under their conditional uniform prior, a distinguishing reveal is worth 319/92 points. A separate archival Odlyzko experiment tests a fixed next-gap classifier on low and remote-height blocks. Its positive log-score gains concern a coarse spacing event, not exact zero recovery. Proofs, exhaustive decision counts, and empirical forecasts are reported separately.

1. The retained object

Let $\mathcal{A} = \{a_1, \ldots, a_{51}\}$ be the reference alphabet, and let $\mathcal{D}$ be its permutations, each identity appearing once. In the card realization, order ranks A,2,...,K and suits ♠, ♡, ♢, ♣, omitting 2♣. The omitted card is an outside marker, not a wild card.

Formula 51 [3] fixes radix 1438, a Hangul decomposition, and a uniform complete-symbol model. Its exact contribution classes have sizes $2, 1, 9, 1, 9, 11, 18$. Let W label those classes. It is a label map, not a sampling probability or the zeta weight in *The Line Game*.

Class	1	2	3	4	5	6	7
Canonical indices	1–2	3	4–12	13	14–22	23–33	34–51
Size	2	1	9	1	9	11	18

Attach the triangular address to the **identity index**, once:

$$m = \frac{n(n-1)}{2} + k, \qquad 1 \le k \le n.$$

With $g = \gcd(n, k)$, $s = n/g$, and $t = k/g$, its polar observation is

$$P(a_m) = \sqrt{s}\, e^{2\pi i t/s}. \tag{1}$$

We test equality through the exact reduced fraction $t/s = k/n$, with $0 < t/s \le 1$, not through rendered pixels. The point determines (s, t) and loses g. Retaining (P, g) recovers (n, k) and then m [5]. The address stays with the identity when the deck moves.

Let σ be signed color, red $+1$ and black -1; let V be the fixed A=1, 10=J=Q=K=10 denomination map; let R_c be natural rank A=1 through K=13. For any token map f, its complete positional profile is

$$\widehat{f}(x) = (f(x_1), \ldots, f(x_{51})).$$

A histogram is a different, coarser observation. A bijective alphabet relabeling preserves every result when the decoder and all maps are transported through its inverse.

2. Joint views and completion information

An observation K recovers a target Q exactly when Q is constant on its fibers, equivalently $\ker K \subseteq \ker Q$ [1]. Telescope 51 gives the first sufficient level along a refinement chain [2]. Here the starting observations need not determine each other.

Theorem 1 (Joint profile count). For maps f, g on an alphabet of N distinct identities used once each, set $c_{uv} = |\{a : f(a) = u, g(a) = v\}|$. Then

$$\ker(\widehat{f}, \widehat{g}) = \ker \widehat{f} \cap \ker \widehat{g},$$

and every attainable complete joint profile has

$$F_{f,g} = \prod_{u,v} c_{uv}! \tag{2}$$

compatible records. Given the profile and dictionary, the minimum worst-case **fixed-length** binary completion is $\lceil \log_2 F_{f,g} \rceil$ bits.

Proof. Equality of both profiles is exactly equality of the joint profile. Positions bearing (u, v) can receive their c_{uv} identities in $c_{uv}!$ ways, independently across cells. A fixed-length b-bit word names at most 2^b records, and enumeration attains the stated bound. $\square$

Under a uniform law on all permutations, the fiber distribution is uniform and its conditional entropy is $\log_2 F_{f,g}$. For a different prior that count supplies an upper bound, not a probability model. The profile and dictionary have their own description costs.

For $J = (W, P)$, exact grouping gives 48 token labels. Its only nonsingleton cells are

$$\{a_6, a_{10}\}, \quad \{a_{15}, a_{21}\}, \quad \{a_{36}, a_{45}\}. \tag{3}$$

They are ($2\heartsuit$, $3\diamondsuit$), ($4\clubsuit$, $6\heartsuit$), and ($10\spadesuit$, $Q\heartsuit$) respectively. Within each pair the polar fraction is 1; their weight classes are respectively 3,5,7. All other joint cells are singletons. Thus every complete J profile has $2!^3 = 8$ lifts, completed by three pair-order bits.

Token view	Labels	Compatible full records per positional profile
W	7	$2!(9!)^2 11!18!$
P	30	50 164 531 200
$J = (W, P)$	48	8
(J, σ)	50	2
(J, V)	50	2
(J, σ, V)	51	1
(J, R_c) or (P, g)	51	1

Color resolves the last two pairs in (3); value resolves the first two. Rank resolves all three. This is sufficiency for this dictionary, not optimization over every possible observation.

3. Sharper recovery sectors

The zero-based permutation rank is

$$\mathcal{R}(x) = \sum_{i=1}^{51} d_i (51 - i)!, \qquad 0 \le d_i \le 51 - i, \tag{4}$$

where d_i counts unused reference identities preceding x_i. Define $\mathcal{D}_k$ to have fixed prefix $a_1, \dots, a_{51-k}$ and every permutation of the final k identities. These are exactly the ranks $0 \le \mathcal{R} < k!$.

Theorem 2 (Sector criterion and sharp bounds). A token profile $\widehat{f}$ is injective on $\mathcal{D}_k$ exactly when f is injective on its final k identities. More generally, its fiber size there is the product of factorials of the label counts **within that suffix**. For the present dictionary:

View	Largest injective $\mathcal{D}_k$	Pair witnessing failure in $\mathcal{D}_{k+1}$
P	$k = 15$	a_{36}, a_{45}: 10♠, Q♡
(P, σ)	$k = 23$	a_{28}, a_{36}: 8♠, 10♠
(W, P, σ)	$k = 45$	a_6, a_{10}: 2♡, 3♢

Proof. The known prefix requires no decoding. Apply (2) to the suffix. A repeated suffix label permits a transposition with equal profile, so injectivity is equivalent to singleton suffix cells.

For (P, σ), the suffix beginning at 29 consists of row 8, row 9, and the first six addresses of row 10. The only repeated polar fractions are 1 at indices 36,45 and 1/2 at 32,50; both pairs have opposite colors. Adding index 28 introduces fraction 1 in black, already present at 36. For P alone, the suffix beginning at 37 has no repetition and adding 36 repeats 45. Finally, globally (W, P, σ) has only the unresolved pair 6,10 from (3). Excluding 6 removes it. Each displayed pair lies in the next suffix and has identical stated labels, proving sharpness. ☐

These are bounds among the specified **factorial sectors**. They do not claim a maximal arbitrary subset or a sharp cutoff among every possible rank interval.

The v0.3 theorem on $\mathcal{D}_{18}$ remains useful: its weight profile is constant, its polar fibers have size two, and polar plus color is injective. The new 23! guarantee contains $23!/18! = 4\,037\,880$ times as many states. Weight profiles cease to be constant across the larger sector; that earlier property has not been extended.

The original 1,566 twelve-place records all lie in $\mathcal{D}_{18}$. Their polar profiles are distinct on that finite archive. A synthetic adjacent-integer collision occurs at record 168, 347272677584420 and its successor: their final two identities are 36 and 45 in opposite orders. The successor is not asserted to be a zero. All original joint-plus-three-bit and polar-plus-color reconstructions pass; the original completion words occur 925 times as `000` and 641 times as `001`. Distinguishing those archived records is weaker than injectivity on the full sector.

4. What a history can add

For deterministic evolution $X_{t+1} = F(X_t)$ and observation K, define

$$H_\ell(x) = (Kx, KFx, \dots, KF^\ell x).$$

Histories refine by truncation, so Telescope's theorem applies to a target recoverable at a declared finite horizon.

Proposition 3 (No refinement under a closed observed update). There exists $\bar{F}$ on $K(X)$ with $KF = \bar{F}K$ if and only if equal current observations give equal next observations. In that case $\ker H_\ell = \ker K$ for every ℓ. If the condition fails, H_1 strictly refines K.

Proof. This is the fiber criterion for the target KF. When the factorization holds, all future observations equal $\bar{F}^t(Kx)$ and are already determined by Kx. Failure exhibits equal initial readings with different next readings. ☐

If every legal action has such an observed update, a policy based only on observed history cannot split initially equal histories. A reveal or an identity-dependent transition changes the available information and must be stated. Randomized games additionally require a transition kernel or a retained random seed.

Telescope's fixed swap illustrates the difference between a total and a history. Both branches have terminal signed sum 1, but their last prefixes are $(1, 0, 1)$ and $(1, 2, 1)$. Their signed second differences, using $(1, -2, 1)$, are $+2$ and -2; their parity paths agree because every ± 1 increment is 1 modulo 2. The signed Pascal and variable-chamber identities from v0.3 remain in the accompanying algebra note. They are algebra on retained data, not an identification with TN Postmaster's analytic source.

Memory 51 can test whether a retained earlier reveal improves a later decision on the **same episode**. Independent uniform new draws cannot be predicted from a previous episode's revealed bit. Replaying already identified archive records measures access or retention; it does not establish prediction of new records.

5. Prospect 51: the conditional experiment

Prospect 51 is the current name of the v0.3 rotor-poker design; its website route is `/area-51/prospect-51/`. The inherited rule identifier R51-0.3 is retained for compatible receipts. Five distinct identities are dealt from the 51-card pack. Hold any subset and replace the rest once, or bank immediately. All five original cards, including discards, are excluded from the replacement pool.

At a selection step with m remaining identities, an affine rotor chooses

$$e = (ad + b) \bmod m, \qquad \gcd(a, m) = 1,$$

from a fresh uniform digit $d \in \{0, \ldots, m-1\}$. Conditional on earlier selections, multiplication by a unit and translation are bijections. Thus every remaining identity is uniform, and the opening distribution is uniform on

$$(51)_5 = 281\,887\,200$$

ordered hands. Prefix-dependent wiring preserves this conclusion if it cannot depend on the current fresh digit. Noncoprime a reaches only $m/\gcd(a, m)$ positions and is a different sampler.

This separates gearing from intervention. Changing valid gearing preserves the distribution. Choosing a hold generally changes it: for r replacements, the conditional domain has $\binom{46}{r}$ equally likely subsets. Poker forgets their order; an ordered bonus does not.

The base gross paytable for a one-point entry is:

Outcome	Gross points	Outcome	Gross points
Jacks or better, one pair	1	Flush	6
Two pair	2	Full house	9
Three of a kind	3	Four of a kind	25
Straight	4	Nonroyal straight flush	50
Royal flush	250	Other	0

Straights allow an ace high in 10–J–Q–K–A or low in A–2–3–4–5, without wraparound; flushes require suit, not color. The independent opening censuses agree on all $\binom{51}{5} = 2\,349\,060$ hands. Banking the opening hand gives base gross expectation $802746/2349060 \approx 0.341731$. This is not an optimal-draw return.

The ordered archive lock remains (10♠, J♦, J♥, K♠, 10♦), tested on the initial slots only, with probability $1/(51)_5$. Its ordinary hand is two pair; rearrangement can preserve that score while losing the lock. The inherited local meter and its reserved bonus remain separate from the base calculations below. Demo points remain separate from shared jbits and Finance holdings.

6. Recover the decision, and price the reveal

Full-state recovery can be more than a decision needs. Let A be a common finite set of legal actions, and let $r(x, a)$ be an expected payoff under a declared transition model. Set $A^*(x) = \arg\max_{a \in A} r(x, a)$.

Theorem 4 (Decision sufficiency). An observation-based policy can attain the full-information optimum in every state exactly when

$$\boxed{\bigcap_{x:K(x)=y} A^*(x) \neq \varnothing \quad \text{for every attained } y.} \tag{5}$$

Proof. One observed value forces one chosen action. That action must be optimal in every compatible state. Conversely choose any action in each nonempty intersection. Randomization cannot repair an empty intersection: to attain every state's maximum, its support must lie in every maximizing set. □

This is a finite decision-theory formulation, consistent with the classical comparison of experiments [7], not a claim to originate that subject. Unlike recovery of a fixed single-valued target, failure need not have a two-state witness: maximizing sets $\{a, b\}, \{b, c\}, \{a, c\}$ intersect pairwise but not jointly. Telescope's pair certificate remains correct for the target maps it actually treats.

Given a finite prior μ, define the best expected payoff using observation K by

$$V(K) = \sum_y \mu(K = y) \max_a \mathbb{E}[r(X, a) \mid K = y]. \tag{6}$$

If L refines K, then $V(L) \geq V(K)$ because an L-policy can ignore the extra field. More explicitly, maximizing separately on refined cells cannot do worse than using the same action in all of them. A reveal with fee c improves this expected account when its conditional value exceeds c. The fee and payoff units are game conventions, not physical information costs.

An exact Prospect witness

Expose only the five $J = (W, P)$ labels of these two ordered hands:

$$H_Q = (\mathrm{A}\heartsuit, \mathrm{K}\heartsuit, \mathrm{J}\heartsuit, 10\heartsuit, \mathrm{Q}\heartsuit),$$

$$H_{10} = (\mathrm{A}\heartsuit, \mathrm{K}\heartsuit, \mathrm{J}\heartsuit, 10\heartsuit, 10\spadesuit).$$

The first four labels are singletons. The fifth is the pair a_{45}, a_{36} in (3). These are the only compatible opening hands, so the uniform opening law gives each conditional probability 1/2. A color reveal in the fifth slot distinguishes them.

Actual hand	Unique optimal hold among all 32	Base gross expectation
H_Q	Keep all five	250
H_{10}	Keep the first four hearts	319/46
Same J reading, no reveal	Keep all five	125

For H_{10}, drawing one card gives one royal completion, eight ordinary flushes, three ordinary straights, nine paying high pairs, and 25 nonpaying cards. Thus

$$\frac{250 + 8 \cdot 6 + 3 \cdot 4 + 9 \cdot 1}{46} = \frac{319}{46}.$$

For H_Q, discarding the queen excludes it from the draw, so the same four-card hold gives $69/46 = 3/2$. An independent C++ enumeration and the unchanged Prospect JavaScript scorer agree on every category count for every hold in both hands. They confirm the unique choices in the table.

The best informed expectation is $(250 + 319/46)/2 = 11819/92$. The blind optimum is 125. Therefore

$$\boxed{\text{value of the color reveal} = \frac{319}{92} \approx 3.467391 \text{ points.}} \tag{7}$$

This is a conditional value for this observation and paytable, not a universal price per bit or an optimal return for the whole game. The entry is already paid, and both hands have no archive bonus, so those terms cancel.

The proposed information mode now has a concrete rule: show the declared joint labels, offer the priced color reveal, commit a hold, then reveal and settle. Before commitment, the score panel and any identity display must not disclose the missing state. Log observation, prior, reveal, fee, hold, and settlement. Memory can supply an earlier reveal on the same episode; it must be counted as available information. This mode is specified and exhaustively analyzed here, not reported as a deployed game or a human trial.

7. Another test on the Odlyzko records

The five supplied tables contain 130,100 rows including the first 100 repeated at high precision. Offsets in the three high blocks must be added to the origins stated in their headers. We subtract stored Decimal values before converting small gaps to floating point. The alphabet and decoder remain fixed.

Representation results

Table	Rows	Transport places	Least k: absolute / listed offset
zeros1, first 100,000	100,000	9	17 / 17
zeros2, first 100	100	12, explicitly rounded	17 / 17
zeros3, near index 10^{12}	10,000	10	23 / 17
zeros4, near index 10^{21}	10,000	8	28 / 15
zeros5, near index 10^{22}	10,000	8	28 / 15

Here k is the smallest integer with every encoded rank below $k!$. The zeros2 row transports nearest twelve-place approximations, not its full thousand-digit strings. Theorem 2 covers the absolute zeros3 records with polar plus color. Adding weight class covers all listed absolute records, including the two higher blocks. All native-scale listed offsets fall inside 18!; decoding an offset still requires its origin to recover the absolute value. Complete replay results accompany the paper.

At the older fixed scale 10^{12}, the absolute-sector sizes instead require $k = 19, 24, 30, 31$ for zeros1,3,4,5. Padding makes all those ranks even. Such an endpoint pattern belongs to the chosen representation, not zero behavior.

The high-precision comparison confirms two one-unit changes under nearest twelve-place rounding: record 58 ends `...420`, where the frozen file has `...421`; record 71 ends `...388`, where the frozen file has `...387`. The frozen fixtures remain the objects replayed by the earlier papers. A separately named corrected branch is supplied. These changes leave their displayed entropy values unchanged and do not alter the theorems.

A fixed forecasting question

Let

$$u_i = (\gamma_{i+1} - \gamma_i)\frac{\log(\gamma_i/(2\pi))}{2\pi}.$$

At the time γ_i is known, the question is whether the next unfolded gap u_i exceeds 1. The predictor uses only u_{i-1}, grouped by fixed boundaries $0.5, 1, 1.5$. Four Bernoulli probabilities, with Laplace smoothing, are trained on the first 60,000 low records. The no-feature baseline uses the same training targets and smoothing. The fitted probabilities are approximately $0.736879, 0.547178, 0.367527, 0.230383$; the baseline probability is 0.465950.

The model is frozen before the following evaluations. Accuracy thresholds forecasts at 1/2; log gain is the mean difference in log scores against that baseline.

Evaluation	Targets	Model accuracy	Baseline	Gain, bits/target
Remaining low records	40,000	61.6300%	53.3975%	0.054999
Entire 10^{12} block, transfer	9,998	61.1322%	53.1406%	0.047865
Entire 10^{21} block, transfer	9,998	60.8022%	53.0806%	0.042179
Entire 10^{22} block, transfer	9,998	60.3821%	52.8406%	0.037852

All 20 consecutive score chunks in each displayed evaluation have positive gain. Each of 199 fixed-seed target-order permutations gives negative gain. Separately specified within-high-block fits also have positive test gain; their full results are reported in the companion review, alongside every forecast. These evaluations overlap and are not independent replications.

The source states guaranteed 10^{-8} ordinate accuracy for zeros3, but only probable, unguaranteed 10^{-6} accuracy for zeros4 and zeros5. Under those stated envelopes, no scored target crosses its threshold. One zeros4 feature is near 0.5. Assigning it adversarially to either adjacent bin leaves the transfer gain at least 0.042136 bits/target. The high-block robustness statement is conditional on the probable accuracy envelope.

This is an archival, within-run locked protocol, not a registered prospective study or a discovery claim about spacing statistics, whose established background includes Odlyzko's work [8]. The result supports this specific preceding-gap observation for this binary task. It supplies neither the exact next ordinate nor a proof of zerohood, completeness, or RH. It also does not attribute predictive power to component weights: the forecast reads the spacing sequence itself.

8. What this version contributes

The three papers now have separate jobs. Formula 51 specifies the alphabet and conventions. Telescope 51 proves recovery along a chain. Project 51 combines observations, identifies sharp factorial-sector limits, and asks whether a distinction changes the best action. Prospect's paid-reveal witness provides an exact connection to incentives; the Odlyzko test provides a separately labeled empirical forecast.

No correction to TN Postmaster v5.1 is required by these results. The Line Game's ambient fiber theorem, positive zero-state distribution, and finite-entropy argument remain unchanged. The new numerical branch records source precision explicitly rather than silently replacing the prior object.

The companion scripts preserve the original 1,566-record audit, reproduce the sector boundaries, enumerate both Prospect decisions independently, and save the forecast protocol, source hashes, predictions, and controls. The exhaustive hold calculation establishes the worked decision; it does not establish full-game economic balance or player performance.

References

1. J. L. Huckstead, *The Line Game / Surprise Is Not Meaning*, v7.5, Zenodo record 22943070.
2. J. L. Huckstead, *Telescope 51: What a Rule Can See*, v0.6 in record 22970030; v0.7 review draft in this package.
3. *Formula 51*, corrected method record, 23 September 2026, with `derive51.py`; concise review edition in this package. Original derivation credited to the Constructive Reasoning Seat.
4. J. L. Huckstead, *Project 51*, v0.3 source, demonstration, and handoff package, 25 September 2026. Its Forge Weights review supplies the retained 48-label joint view and three residual pairs. The separately retrieved v2.2.1 website delta establishes the Prospect name and unchanged R51-0.3 rules.
5. J. Huckstead, *Divisor-Band Bijections and Totient Shells in the Triangular-Fractional Grid*, doi:10.5281/zenodo.21440877; *The Polar Shell Renderer*, doi:10.5281/zenodo.21280464. Address conventions retained from v0.3.
6. J. Huckstead, *TN Postmaster*, Volume I, v5.1, Technical Dossier (163A.6)–(163A.9), record 22844194. Frozen source.
7. D. Blackwell, "Equivalent Comparisons of Experiments," *Annals of Mathematical Statistics* **24** (1953), 265–272, doi:10.1214/aoms/1177729032. Classical decision-theory context; the finite statements used here are proved above.
8. A. M. Odlyzko, Tables of zeros of the Riemann zeta function; "On the distribution of spacings between zeros of the zeta function," *Mathematics of Computation* **48** (1987), 273–308, doi:10.1090/S0025-5718-1987-0866115-0.
9. Bicycle/United States Playing Card Company, Basics of Poker, accessed 26 September 2026. Familiar hand categories; the 51-card pack, paytable, information mode, and archive lock are the declared Project conventions.

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