Project 51

Reading the Record Across Rules

Joint observations, factorial fibers, and recoverable play

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Abstract

An ordered record can survive a change of alphabet while a measurement loses distinctions. Project 51 asks which observations recover them and what a retained history adds. We construct a 51-symbol alphabet and its decoder explicitly. Exact component-weight class and reduced polar address together leave eight compatible arrangements per complete positional profile; three binary choices complete recovery. On the entire sector of permutation ranks below 18!, polar point and signed color recover the record. Both decoders are checked on 1,566 stored ordinate approximations. The published Telescope pair has equal resolution summaries but different labeled winners, and equal parity paths but opposite signed second differences. Fiber counting and a deterministic history criterion connect these examples without requiring card faces. A prospective Memory 51 protocol separates retained information, prediction, and reveal cost. A rotor-poker extension distinguishes invertible selection, player intervention, and order-sensitive jackpots. The results concern recovery under declared conventions; predicting new data remains a separate experiment.

A:	J♡ K♠ 10♢	14.134725141735	king at P2
B:	J♡ 10♢ K♠	14.134725141734	king at P3

Only the final three positions are shown; the first 48 are unchanged. These small tiles are identity notation, at reading scale: rank and suit remain visible without a full card face. Both weight profiles agree. The decoder changes by one last-place unit; Telescope selects a different labeled position.

1 One record, several ways to read it

Lay out 51 distinct things. Their inventory can stay fixed while their order changes. A reader who sees only a weight attached to each position may miss that change; a reader who also sees color or an address may recover it. The object is the ordered record. An observation is a declared rule for reading it. Recovery asks whether that reading determines a specified target: the whole order, its encoded integer, or one feature of a later game.

This is Project 51's working question: keep the object, change what can be seen, and ledger the distinction that each rule retains. A picture, score, or familiar game gives a useful observation when its input, output, and inverse are clear. The present paper supplies those definitions before using the earlier work.

Let $\mathcal{A} = \{a_1, \ldots, a_{51}\}$ be an ordered alphabet. A state $x = (x_1, \ldots, x_{51})$ contains each identity once; $\mathcal{D} = \mathrm{Perm}(\mathcal{A})$ is the set of all such states. The card realization assigns the reference identities in rank order A,2,. . .,K and suit order S,H,D,C, omitting 2♣. That card remains outside as marker I. The index m names an identity, not its changing position in a shuffled deck.

For any observation $K : X \to Y$ and target $Q : X \to Z$, recovery means that a function h exists with $Q = h \circ K$. The *fiber* of an observed value is the set of states that give that same value. Recovery is possible exactly when Q is constant on each fiber: equal readings must have equal target values. Conversely, that common target value defines h. We write $\ker K = \{(x, x') : K(x) = K(x')\}$ and $K \preceq L$ when $\ker L \subseteq \ker K$; L then refines K.

The Line Game | Surprise Is Not Meaning uses this criterion to separate measurement from the question put to it [1]. *Telescope 51* shows that along a finite refinement chain, once a target becomes recoverable it stays recoverable. For a nonconstant target recovered at the finest level, the first sufficient level is the latest separation level among target-distinct pairs [2]. Here we join observations that need not determine one another, count the ambiguity they leave together, and extend the same question to a history of play.

Where the 51 identities and their weights come from

Formula 51 fixes $N_0 = 1438$, the original tabulated-prefix count, as a radix [3]. Later record files may be longer; this construction keeps that radix. Render digit $d \in \{0, \dots, 1437\}$ as Unicode syllable `U+AC00`$+d$. Its canonical Hangul decomposition is specified by

$$d = ((21L) + V)28 + T, \qquad 0 \le V \le 20, \quad 0 \le T \le 27.$$

Here $T = 0$ denotes an absent trailing component. The inventory contains 3 leading components, all 21 vowels, and all 27 visible trailing components: $3 + 21 + 27 = 51$.

Choose a complete syllable uniformly from this inventory. Reading its components along the conditional tree L, then V, then T or END, assign component c the expected contribution

$$w(c) = \frac{1}{N_0} \sum_{d:\, c \text{ occurs in } d} \log_2 \frac{N(\text{parent of } c \text{ in } d)}{N(\text{child after } c \text{ in } d)}. \tag{1.1}$$

$N(\cdot)$ counts inventory syllables compatible with the indicated prefix; END records the absent trailing branch. The chain rule gives

$$\sum_{c \text{ visible}} w(c) + w(\text{END}) = \log_2 1438.$$

Sort visible components by decreasing contribution, breaking exact ties by code point, then assign them to the reference identities. The exact prime-log calculation reproduces the published twelve-decimal sorting convention and the following classes. $W(a_m)$ labels the class; its number is not the contribution in bits.

Table 1: The retained class map W. Ranges are inclusive.

Class	1	2	3	4	5	6	7
Canonical indices	1–2	3	4–12	13	14–22	23–33	34–51
Class size	2	1	9	1	9	11	18

The decomposition, probability model, and sorting convention are inputs. Ordinate values do not enter the weight calculation. Since $1438 = 51 \cdot 28 + 10$, trailing components $T_{10}, \dots, T_{27}$ share the 51 complete contexts and form an exact 18-member tie; comparison with the other contributions places it last at this radix. Neither 51 nor this ordering is inferred from a hidden law of the zeros. A different source with the same declared radix gives the same component model.

The alphabet can change without changing the experiment

For any bijection $b : \mathcal{A} \to \mathcal{A}'$, relabel a record coordinatewise by $B(x) = (b(x_1), \ldots, b(x_{51}))$. Transport a token observation by $f' = f \circ b^{-1}$ and a record decoder by $D' = D \circ B^{-1}$. Then

$$\widehat{f}'(Bx) = \widehat{f}(x), \qquad D'(Bx) = D(x).$$

A transition transports as $F' = B \circ F \circ B^{-1}$, so its histories correspond as well. Cards, letters, or distinct physical marks can therefore realize the same object. The dictionary and rules move with the labels; assigning new meanings to the labels is a new experiment.

2 Joint observations and the factorial account

For a token observation $f : \mathcal{A} \to Y$, write

$$\widehat{f}(x) = (f(x_1), \ldots, f(x_{51})). \tag{2.1}$$

This is the *complete ordered profile.* An unordered histogram is a different, coarser record.

Theorem 1 (Joint observations and residual arrangements)**.** *Let f and g be observations of a finite alphabet of N distinct identities, each used once per record. Set*

$$c_{uv} = |\{a : f(a) = u,\ g(a) = v\}|.$$

Then the joint observation $J = (\widehat{f}, \widehat{g})$ satisfies

$$\ker J = \ker \widehat{f} \ \cap\ \ker \widehat{g}. \tag{2.2}$$

Every attainable complete joint profile has exactly

$$F_J = \prod_{u,v} c_{uv}! \tag{2.3}$$

compatible records. A target Q is recoverable from J exactly when it is constant on those fibers. Given a joint profile and the shared dictionary, the minimum worst-case length of a fixed-length binary completion code for the full record is $\lceil \log_2 F_J \rceil$.

Proof. Two records have the same joint observation exactly when both observations agree, proving (2.2). Fix a joint profile. Its positions carrying label (u, v) must be filled by the c_{uv} identities with that label. They can be assigned in $c_{uv}!$ ways, independently for each cell. Multiplication gives (2.3); $0! = 1$ handles empty cells. The target statement follows from the fiber criterion proved in Section 1. Finally, b binary symbols distinguish at most 2^b possibilities, while enumerating the F_J compatible records constructs a completion code of length $\lceil \log_2 F_J \rceil$. $\square$

This answers why factorial groups occur in the present object. A group of d identities carrying the same observed label hides $d!$ arrangements. Refinement splits that group into cells and replaces $d!$ by a product of smaller factorials. The identity $d! = d(d-1)!$ records the first identity choice followed by the remaining choices. A singleton class contributes $1!$; an empty cell contributes $0!$. Both factors equal one, but their class sizes differ. The same counting supports the mixed-radix permutation decoder used below.

For the weight profile alone, Table 1 gives

$$F_W = 2!(9!)^2 11!18! = 67\,305\,847\,895\,575\,919\,030\,981\,099\,520\,000\,000. \tag{2.4}$$

Under a uniform distribution on all $N!$ permutations, the conditional distribution within each profile is uniform, so $H(X \mid J) = \log_2 F_J$ bits [8]. For an arbitrary source distribution this becomes an upper bound; a count alone does not choose the distribution. Completion bits are conditional on the profile and dictionary, whose own description costs remain in the full account.

3 A fixed address for each of the 51 identities

TFG's flattened triangular address is [4]

$$m = \frac{n(n-1)}{2} + k, \qquad 1 \le k \le n, \tag{3.1}$$

with a unique inverse for every positive integer m. Apply it once to the canonical identity index. The resulting address stays with the token during every shuffle. It is never assigned from the token's current deck position.

Put

$$g = \gcd(n,k), \qquad s = n/g, \qquad t = k/g, \qquad P(a_m) = \sqrt{s}\, e^{2\pi i t/s}. \tag{3.2}$$

This is the denominator-collapse map of the polar renderer [5]. We compute it using the exact reduced fraction t/s. Under the declared orientation and $0 < t/s \le 1$, that fraction uniquely identifies the exact point. Display rounding is not part of the equality test.

The point retains (s,t) and omits g. Thus $(3,2),(6,4),(9,6)$ give the same point, while (P,g) recovers $(n,k) = (gs, gt)$ and hence m. A rotation or nonzero scale applied to every rendered point is invertible and leaves these fibers unchanged; it cannot restore the omitted multiplier. The address is a declared view of the retained identity. Keeping g repairs this reduction without changing the underlying token.

The supplied Forge Weights review proposed combining this point with W [7]. Its finite result is reproduced here and extended to the card observations and source sector below. Let $J = (W,P)$ at token level. Direct grouping of the 51 fixed addresses gives Table 2.

Table 2: Observations on the full 51-identity alphabet. The last column counts full deck arrangements compatible with one complete positional profile.

Token observation	Labels	Compatible arrangements
W	7	$2!(9!)^2 11!18!$
P	30	50 164 531 200
$J = (W,P)$	48	8
(J,σ)	50	2
(J,V)	50	2
(J,σ,V)	51	1
(J,R_{card})	51	1
(P,g)	51	1

Here σ is $+1$ for red and -1 for black; V is the Telescope value convention A=1, ranks 2–9 at face value, and 10,J,Q,K all valued 10; R_{card} is natural rank A=1 through K=13. This fixed V is a token observation, not the contextual soft-ace valuation of a blackjack hand.

Proposition 2 (The three residual pairs)**.** *The only nonsingleton J-classes are*

$$\{a_6, a_{10}\}, \qquad \{a_{15}, a_{21}\}, \qquad \{a_{36}, a_{45}\}. \tag{3.3}$$

Consequently every complete joint profile has eight lifts to $\mathcal{D}$. Three binary choices of pair order recover the entire deck.

Proof. Within each index range of Table 1, reduce k/n from (3.1). The only repeated fractions within a range are the three pairs in the display below. All other (W, P) cells are singletons. Equation (2.3) gives $2!2!2! = 8$. Reading each pair's two occurrences in their chosen order constructs the unique compatible deck. □

2♡ 3♢	4♣ 6♡	10♠ Q♡
$(3,3), (4,4)$	$(5,5), (6,6)$	$(8,8), (9,9)$
value separates	color or value separates	color separates

Each pair has one joint label and two possible orders. The addresses above belong to the identities, never to the current card positions.

Color resolves the last two pairs; value resolves the first two. Their combination resolves all three. Natural rank resolves each pair on its own. These are different sufficient collections of observations. For example,

$$W \;\preceq\; (W, P) \;\preceq\; (W, P, \sigma) \;\preceq\; (W, P, \sigma, V)$$

is a refinement chain with residual counts $F_W, 8, 2, 1$. Along this chain the full-identity target is first recoverable at the last level, with 2♡/3♢ as its persistent witness. A chain using natural rank instead has a different threshold. This applies Telescope's theorem without mistaking one chosen chain for an optimum over all possible rules.

4 An exact sector and the 1,566-record audit

The inherited zero-based Lehmer rank of a deck is

$$\mathcal{R}(x) = \sum_{i=1}^{51} d_i (51 - i)!, \qquad 0 \le d_i \le 51 - i, \tag{4.1}$$

where d_i counts unused reference identities preceding x_i. At the declared decimal scale 10^{12}, each stored ordinate approximation supplies an integer R, which is unranked by (4.1). This represents one stored decimal per deck. The reference order and scale are part of the decoding convention.

The full record behind the opening comparison stores 14.134725141735. Thus $R = 14\,134\,725\,141\,735$, and unranking gives branch A:

A♠ A♡ A♢ A♣ 2♠ 2♡ 2♢ 3♠ 3♡ 3♢ 3♣ 4♠ 4♡ 4♢ 4♣ 5♠ 5♡

5♢ 5♣ 6♠ 6♡ 6♢ 6♣ 7♠ 7♡ 7♢ 7♣ 8♠ 8♡ 8♢ 8♣ 9♠ 9♡ 9♢

9♣ Q♢ K♡ 10♡ K♢ Q♡ Q♠ J♠ 10♣ J♣ Q♣ K♣ 10♠ J♢ J♡ K♠ 10♢

The outside marker is 2♣; it does not enter the permutation rank. Branch B swaps only the final K♠, 10♢, decreasing R by one and decoding to 14.134725141734. Both complete weight profiles agree. The integer chooses an order inside the freedom left by the weights; the weights did not generate the integer.

Let $\mathcal{D}_{18}$ contain every deck with canonical prefix $a_1, \ldots, a_{33}$ and an arbitrary permutation of $E = \{a_{34}, \ldots, a_{51}\}$ afterward. Equivalently, $\mathcal{D}_{18}$ contains exactly the ranks $0 \le R < 18!$.

Theorem 3 (Recovery throughout the terminal sector)**.** *On $\mathcal{D}_{18}$ the weight profile is constant; each attainable polar profile has exactly two compatible decks; and the complete polar-and-color profile is injective. Therefore P together with σ recovers every rank in $[0, 18!)$, given the declared sector and reference dictionary.*

Proof. All identities in E have weight class 7, so permutations of E preserve the full weight profile. The addresses of E are

$$(8,6),(8,7),(8,8); \qquad (9,1),\ldots,(9,9); \qquad (10,1),\ldots,(10,6).$$

Reducing these 18 fractions leaves only one duplicate: $(8,8)$ and $(9,9)$, belonging to a_{36} and a_{45}. Each full polar profile therefore has two lifts in the sector. These two identities are 10♠ and Q♡, of opposite colors. The pair (P,σ) distinguishes every member of E, and the prefix is already fixed. Positionwise inversion recovers the deck, then (4.1) recovers its integer. □

The domain declaration matters. Globally, (P,σ) has 36 token labels and 1 105 920 compatible arrangements per profile. Its injectivity in Theorem 3 comes from the specified sector. Conversely, the theorem covers all $18! = 6\,402\,373\,705\,728\,000$ sector states; it does not depend on enumerating those states or on zerohood.

The supplied archive contains 1,566 strictly increasing stored records, all with $R < 18!$. The initial comparison protocol fixed the weight, point, joint, and point-plus-gcd views before their source counts were computed. The present extension adds the polar-and-color decoder of Theorem 3. These records had already been used in earlier rounds: the following audit measures exact retention on that archive, not performance on untouched data.

Table 3: Complete profiles on the 1,566 stored records. The last column counts equality with the profile of the synthetic neighboring integer $R+1$.

Observation	Distinct archive profiles	Unchanged under $R+1$
W	1	1,566
P	1,566	1
(W,P)	1,566	1
(P,g)	1,566	0
(P,σ)	1,566	0

Polar point alone already separates the actual archived records; on that restricted finite domain it therefore also determines every target of the record, given a lookup table. Weight adds no further distinct profiles within this sector. That archive separation is weaker than injectivity on the entire sector. The sole adjacent-integer joint collision occurs at record 168:

$$R = 347272677584420, \qquad R+1 = 347272677584421.$$

The last two positions exchange identities 36 and 45. Color or the gcd multiplier distinguishes them. The synthetic neighboring integer is not asserted to name another zero.

Two interventions identify the operative fields on every source deck. Exchanging identities 34 and 35 preserves the complete weight profile and changes the polar profile. Exchanging identities 36 and 45 preserves both weight and polar profiles, while changing color and gcd observations. The interventions act on named identities wherever they occur, rather than on fixed positions.

Every source deck and integer was reconstructed from its joint profile and the three pair-order bits of Proposition 2. In the order of (3.3), the words were `000` for 925 records and `001` for 641. Only the last bit varies because the first two pairs are in the fixed prefix. A second decoder recovered every source integer directly from the polar-and-color profile under the sector declaration. The profile and dictionary carry the rest of the information in both accounts.

5 The Pascal connection already present in Telescope

The printed branches also let a reader reconstruct Telescope's witness without another paper [2]. Take each consecutive triple as one round, with its three positions labeled P1, P2, P3. Among tied leaders, maximize V, then natural rank, then signed color, then suit in the order S,H,D,C. A stage never restores an eliminated candidate. Record the first stage leaving one leader, using letters V,R,C,S. For both branches the 17-round word is

`SCCV SSCV SSCV SCRRR.`

The opening comparison shows the final triple. All three values are 10; natural rank selects the king, hence P2 in A and P3 in B. Both candidate-count trajectories are $(3, 3, 1, 1, 1)$ and earlier triples agree. The whole resolution word and candidate-size ledger therefore fail to recover the labeled winner. The tie-only decision rule is distinct from the observation-refinement theorem.

The same pair has equal terminal color total and different penultimate prefixes. Set

$$S_0 = 0, \qquad S_j = \sum_{i=1}^{j} \sigma(x_i). \tag{5.1}$$

Because the active alphabet has 26 red and 25 black identities, $S_{51} = 1$ for every ordering. Yet the full path recovers the color word by $\sigma(x_j) = S_j - S_{j-1}$. The outside black marker would restore total zero. The endpoint and the full path answer different recovery questions.

For the published branches, the last three prefixes and their signed Pascal readings are

	(S_{49}, S_{50}, S_{51})	parity	$S_{49} - 2S_{50} + S_{51}$
A	$(1, 0, 1)$	$(1, 0, 1)$	$+2$
B	$(1, 2, 1)$	$(1, 0, 1)$	-2

(5.2)

The row $(1, -2, 1)$ is the degree-two signed Pascal row. Its value here is exactly $\sigma(x_{51}) - \sigma(x_{50})$. The parity view merges the branches; the signed difference separates them. Indeed $S_j \equiv j \pmod 2$ for *every* ordering, since $+1$ and -1 are equal modulo two. This alternating parity pattern is forced by the observation rule. This is a concrete continuation of Telescope's own witness, with the same cards and prefixes.

TN v5.1 uses the fuller signed Pascal structure [6]. For indices $j, r = 0, \dots, N$, put

$$U_{jr} = \begin{cases} (-1)^j \binom{r}{j}, & j \le r, \\ 0, & j > r, \end{cases} \qquad M_{rs} = \binom{r+s}{r}. \tag{5.3}$$

Then

$$U^2 = I, \qquad U^T U = M, \qquad d^T M d = \|Ud\|_2^2 \tag{5.4}$$

for every real column vector d. These are algebraic identities independent of the data assigned to d.

For completeness, the (j, r) entry of U^2 is

$$\binom{r}{j} \sum_{\ell=0}^{r-j} (-1)^\ell \binom{r-j}{\ell},$$

which equals 1 when $j = r$ and 0 otherwise. Also $(U^T U)_{rs} = \sum_j \binom{r}{j}\binom{s}{j} = \binom{r+s}{r}$ by the binomial convolution. The quadratic identity follows by multiplication. The companion runner checks these matrices in exact integer arithmetic through degree 12.

Reducing signed Pascal coefficients modulo two gives the familiar parity pattern, but discards their signs. Equation (5.2) shows a concrete distinction lost by a modulo-two reading of the path. Project 51 can keep the signed vector, inspect its parity pattern, and apply the invertible transform U as separate operations. A scalar energy such as $\|Ud\|^2$ is a further observation and need not recover d. Applying the matrix to a card path also does not identify that path with TN's analytic source.

6 When the base changes

Three different choices of scale already coexist. The historical component construction fixes radix 1438. The permutation decoder uses the changing digit ranges in (4.1). TFG permits a chamber-dependent alphabet $\{1, \ldots, n\}$. Calling all three a "base" can hide which state and inverse are being changed.

Within a known TFG chamber n, observe k through the reduced denominator

$$s = \frac{n}{\gcd(n, k)}. \tag{6.1}$$

For each divisor $s \mid n$, the class consists of $k = (n/s)t$ with $1 \le t \le s$ and $\gcd(t, s) = 1$. Its size is $\varphi(s)$, including $\varphi(1) = 1$. Keeping n, s, and the rank of t among those reduced numerators recovers k.

Chamber	Divisors in order	Corresponding class sizes
20	$1, 2, 4, 5, 10, 20$	$1, 1, 2, 4, 4, 8$
51	$1, 3, 17, 51$	$1, 2, 16, 32$

For permutations of all n states, a fixed denominator-class profile consequently has $\prod_{s|n} \varphi(s)!$ compatible arrangements. If k is sampled uniformly, its class probability is $\varphi(s)/n$. Those probabilities belong to this declared model. Equal class sizes do not identify distinct divisor labels.

This gives a precise option for testing variable alphabets: retain the chamber or the rule determining it, and provide the matching inverse. A chamber sequence that is available only to the encoder is extra information. These alternative partitions are available to Project 51 without replacing the inherited seven classes by implication. The present experiments retain the original alphabet throughout.

7 What play and memory can add

Static observations can be combined across fields. They can also be accumulated across time. Let a deterministic rule evolve a full state by $X_{t+1} = F(X_t)$, and let a player see $K(X_t)$. The finite history map is

$$H_\ell(x) = (K(x), K(Fx), \ldots, K(F^\ell x)). \tag{7.1}$$

These maps form a refinement chain because truncation recovers $H_{\ell-1}$. Telescope's threshold theorem therefore applies to a target recoverable at a declared finite horizon. Waiting helps only if later observations split a relevant earlier fiber.

Proposition 4 (When repeated observation cannot refine the state)**.** *There exists a deterministic observed transition* $\bar{F} : K(X) \to K(X)$ *satisfying*

$$K \circ F = \bar{F} \circ K \tag{7.2}$$

if and only if $K(Fx) = K(Fx')$ *whenever* $K(x) = K(x')$*. Under this condition,* $\ker H_\ell = \ker K$ *for every* $\ell \geq 0$*. If the condition fails,* H_1 *strictly refines* K*.*

Proof. Condition (7.2) is the fiber criterion for the target $K \circ F$. When it holds, each future observation is $\bar{F}^t(Kx)$, so Kx determines the entire history. Conversely the history retains Kx as its first entry, yielding equality of kernels. Failure provides two states with equal initial observations and unequal next observations, which H_1 separates. □

The transition may use a distinction concealed from the player and make its effect visible later. For a small example, take states $\{0, 1, 2\}$, observations $(0, 0, 1)$, and transitions $F(0) = 0, F(1) = 2, F(2) = 2$. The first two states initially agree, while their one-step histories are $(0, 0)$ and $(0, 1)$. All three states are then distinguished. This example demonstrates the mechanism; no unseen card-game trajectory is asserted.

The same reasoning extends to declared actions. If each allowed action has an observed update of the form (7.2), a policy using only observed history cannot split initially equal observations: equal histories select equal actions and remain equal. If an action consults identity, reveals color, or records a previously omitted arrival, its information use must be included in the model. In a randomized rule the seed or transition kernel must also be specified.

This gives the game suite a research structure. Invariant supplies a retained arrangement and continuation rule. Telescope supplies explicit observation thresholds. Blackjack supplies value and payoff conventions, while Memory 51 can retain earlier reveals and test whether they improve a subsequent decision. Familiar rules provide inspectable maps; their consequences follow from the maps and transitions actually declared.

A specified next experiment

One Memory 51 target is the remaining sector bit

$$B(R) = \mathbf{1}\{a_{36} \text{ occurs after } a_{45} \text{ in the decoded deck}\}. \tag{7.3}$$

On $\mathcal{D}_{18}$, each polar profile admits exactly one deck for each value of B; a color reveal at an ambiguous position selects the deck. This is structural ambiguity on the sector. The already studied archive is a different domain: its distinct polar profiles permit a lookup of B. Replaying those records can test memory or decoding, but cannot by itself test prediction on new records.

Two controls give the next experiment a clear meaning. Independent uniform draws from $\mathcal{D}_{18}$ make B conditionally fair given the current polar profile: swapping identities 36 and 45 pairs the two cases. Earlier independent draws cannot improve that forecast. A memory task retains an earlier reveal within an episode and later asks for the same bit while hiding the cards as illustrated below. Its comparison is retained history versus the same current display without that history. This tests access and retention. A claim about predicting new source records needs a separate, fresh sequence and a declared source model.

Earlier reveal	Same current display	Retained bit
10♠ Q♡	two face-down slots	$B = 0$
Q♡ 10♠	two face-down slots	$B = 1$

This is a retention-task illustration, not a participant result. The earlier order supplies the bit that the current display omits.

For either task, fix the sampling rule, permitted fields and histories, predictor, baseline, and scoring schedule before play. The reference dictionary remains public. A full source integer, an identifying archive lookup, or a current identity reveal would answer this target directly and must be accounted for if available. Known rules alone still leave the two sector possibilities; secrecy of the convention is not the source of that ambiguity.

Let q_t and $q_{0,t}$ be strictly positive forecast distributions committed before the scored answer is revealed. An example account is

$$G_t = \log_2 \frac{q_t(B_t)}{q_{0,t}(B_t)} - c_t. \tag{7.4}$$

The first term is a difference of predictive log scores in bits. The nonnegative reveal charge c_t is converted into those units by an explicit game convention. It is not a measured energy cost. A bit purchased before the commitment is observed information and cannot also earn an unassisted prediction score on that same question. Its cost can be logged now and its benefit evaluated on later, predeclared queries.

Report the complete sequence, both information budgets, and which history access differs. The baseline shares all inputs except the feature being tested. Within-fiber swaps audit which fields a predictor uses, but predictive generalization requires data unused in choosing the rule. The 1,566-record audit establishes the decoder and its distinctions; the memory and prediction trials remain prospective.

8 Rotor 51: chance, intervention, and an ordered target

The same identity may be printed as 8♠ or as its component symbol. Poker first uses the familiar rank and suit dictionary; a click in the accompanying demo reveals the exact paper symbol. This is the transported relabeling of Section 1, not a new assignment of card meanings. The sampler, score, holds, and jackpot all continue to act on the retained identities. The underlying component weights are not used as sampling probabilities.

Let the inventory determine the moduli

Rotor 51 is a new declared rule on five distinct identities, rather than a replay of Telescope or a randomization of the stored ordinate. Its opening state space has

$$(51)_5 = 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47 = 281\,887\,200$$

ordered hands. Begin with the canonical 51-identity list. For rotor $j = 1, \dots, 5$, let $m_j = 52 - j$ be the size of the remaining inventory, draw d_j uniformly from $\{0, \dots, m_j - 1\}$, and select and remove the identity at zero-based index

$$e_j = (a_j d_j + b_j) \bmod m_j, \qquad \gcd(a_j, m_j) = 1. \tag{8.1}$$

The wiring and offset may depend on the observed prefix but not on the current fresh digit. In the demo, each offset combines a ring setting, a triangular offset T_j, and a carry from earlier selections. This is a declared design convention, not a probability law extracted from T_j.

Proposition 5 (Fair gearing and lost positions)**.** *Under independent uniform input digits, any such coprime wiring produces the uniform distribution on ordered distinct five-identity hands. A general affine map modulo m with $g = \gcd(a, m)$ reaches exactly m/g positions, with g input digits per reachable position.*

Proof. Condition on the preceding selections. Multiplication by a unit modulo m_j and translation by b_j are bijections, so each remaining identity has conditional probability $1/m_j$. Multiplying the five conditional probabilities gives $1/(51)_5$. For the general map, $ad \equiv e - b \pmod{m}$ is soluble exactly when g divides $e - b$, and then has g solutions modulo m. □

The distinction answers the modulus question without inventing extra identities. Two mixed-radix dials $u \in \{0, 1, 2\}$ and $v \in \{0, \dots, 16\}$ give $i = 17u + v \in \{0, \dots, 50\}$ bijectively, naming a_{i+1}. Either dial alone loses distinctions; together they identify the token. Five independent mod-51 reels would instead permit repeated identities, with state space 51^5. That is a different game, not the implemented poker deal. The five live depletion rotors use the actual remaining inventory. Three- or seven-card variants would require new scoring and new counts.

Player choice changes the conditional experiment

After the first five identities are exposed, the player may hold any subset and replace the others once. All five initial identities, including discards, stay out of the replacement pool. Replacing r positions therefore uses moduli $46, 45, \dots, 47 - r$. Conditional on the initial hand and chosen hold, the $\binom{46}{r}$ possible replacement subsets are equiprobable under the same uniform-digit model. The hold changes that domain and hence the score distribution. Merely changing one valid gearing for another does not change the opening distribution. This separates an invertible change of representation from a strategic intervention.

The demo uses the declared five-card ranking, with aces high or low only in A–2–3–4–5. Its one-point entry has gross awards $1, 2, 3, 4, 6, 9, 25, 50, 250$ for jacks-or-better, two pair, trips, straight, flush, full house, four of a kind, nonroyal straight flush, and royal flush respectively; other hands pay zero. These amounts are design choices, not a claimed return under optimal play. Exhaustive enumeration gives $\binom{51}{5} = 2\,349\,060$ opening subsets, including four royals. Those opening counts do not apply unchanged after an adaptive hold. The conditional-odds panel enumerates the actual $\binom{46}{r}$ completions.

Poker forgets order. Each distinct five-card subset has $5! = 120$ ordered realizations; its score is coarser still. The second target retains order: the *archive lock* is the published record's final five identities,

10♠ J♢ J♡ K♠ 10♢.

Swapping its first two cards preserves the poker hand and award but loses this target. Under the uniform opening model the exact lock has probability $1/(51)_5$. It is tested only on the initial deal; a later redraw cannot create eligibility. This is a concrete witness that an unordered poker reading does not recover an ordered jackpot target.

The local demo meter starts at 51 points and increases by 0.01 per paid deal. An initial lock reserves that incremented meter for settlement and resets the meter to 51; its award is added to the eventual hand award. Illustrative jackpot examples cannot debit or credit the bank. Points are isolated from shared jbits and Finance holdings. The rotor styling is not Enigma encryption; the browser demo is not a certified random source, a global jackpot service, or a proof of economic balance. Its purpose is to make each map, intervention, and information loss inspectable.

9 What transfers, and what the reader can verify

The earlier papers contribute different parts of this object. Their relationship is precise enough to state without making one source stand for another.

Source	What Project 51 carries forward
Line Game	The fiber criterion, exact ordered record, decoder, and separation of a measurement from its interpretation.
Telescope 51	Recovery along refinements; the published pair and its tie-only decision protocol.
Formula 51	The fixed radix, component contribution model, identity dictionary, and seven exact classes.
TFG and polar renderer	A unique triangular address, then an explicit reduction that omits the gcd multiplier.
TN Postmaster	The signed Pascal involution and Gram identity; their algebra is independent of the vector supplied to them.
Project 51	Joint fibers, completion codes, sector recovery, a history criterion, and a declared rotor-poker extension.

TN also explains the analytic fold behind the Line Game. With $T_z = z(z+1)/2$,

$$q(s) = s(1-s) = \tfrac{1}{4} - (s - \tfrac{1}{2})^2 = -2T_{s-1}, \qquad q(s) = q(1-s).$$

Keeping the complex q determines the unordered reflection pair through $s^2 - s + q = 0$. Taking a real part, scalar weight, or energy is an additional observation with its own fibers. The Line Game's scalar collision is on its stated ambient domain; it is not a collision established on the actual zero locus. Project 51's component class W is a different map. What transfers between these settings is the recovery criterion, with the domain and target stated anew. A card-path vector fed to TN's Pascal matrix does not become TN's completed analytic source.

For Area 51, the resulting rule card has a small set of obligations: identify the current state and target, show which fields are visible, declare the transition and available history, and account for each reveal. Invariant provides an arrangement and its deterministic continuation; Telescope exposes observation depth; Blackjack makes value depend on a hand and its rule; Memory can compare retained and hidden reveals; Rotor separates gearing, a hold decision, and an ordered target. These interfaces are places to test the declarations. No game outcome alone establishes source meaning, a zero law, or a physical information cost.

The companion package includes the exact record above, unchanged publication fixtures, all 1,566 stored records, a complete token dictionary, and a standard-library verifier. Its 65 named checks cover exact class grouping, the factorial product rule on a small enumerated alphabet, both source decoders, the controlled swaps, the Telescope witness, signed Pascal identities, and the chamber partitions. Additional checks reproduce the worked example and the behavior under nonnumeric relabeling. A separate engine audit enumerates all 2,349,060 opening hands and tests the rotor maps, hold masks, marker exclusion, and one-time settlement. Proofs establish the general claims; finite replays check this realization.

The 48-label joint view and its eight lifts originate in the retained Forge Weights review [7]. The v0.2 recovery results are retained. Version 0.3 adds compact identity notation and the separately declared Rotor 51 rule; it does not promote that design into a theorem about the source. It reports no new zero certification or prospective learning result. The record stays available for further questions: factorials count the distinctions a view leaves unresolved, another field can recover them, and a remembered observation can make earlier information available to a later decision.

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[7] *Forge to Area 51: Retaining the Object While Testing a View*, research checkpoint, 24 September 2026. Supplied `Forge_Weights_Review_2026-09-24.zip`, §§1–3 and finite replay. The unchanged review is included with this paper's computational materials.

[8] C. E. Shannon, "A Mathematical Theory of Communication," *Bell System Technical Journal* **27** (1948), 379–423, 623–656. Author text, reprinted with corrections.