Volume I: Exact Geometry and Symbolic Foundations

A Lead Companion on the 45° Cone, Pell Curvature, and Addressable Number Structure

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Abstract

This lead companion rewrites the earlier *Scalar Emergence and Symbolic Foundations* volume into the part that survives independent checking: exact geometry and exact address structure. The original project treated $\pi/\sqrt{2}$ as a broad physical scalar. This revision deliberately narrows the claim. It keeps the self-contained mathematics: a 45° cone with two independent sphere-packing operators, a Pell curvature ladder in $\mathbb{Z}[\sqrt{2}]$, and a divisor-band address rule for rational messages. It removes or quarantines physical, acoustic, market, and constant-governing interpretations that the exact mathematics does not support.

The governing theme is layer discipline. Geometry may recover a volume ratio. Number theory may recover an address. A public convention may recover a letter. Meaning and physical interpretation require additional evidence. The result is not a unified scalar physics. It is a cleaner monograph: exact identities, stated provenance, symbolic verification, and a firewall between structure and interpretation.

1 What changed, and why

The earlier Volume I presented a broad symbolic program: constants, light, mass, atomic structure, resonance, and observer geometry were tied together by the Huckstead Scalar

$$a_H = \frac{\pi}{\sqrt{2}}.$$

That document was valuable as a research record, but it blurred three very different kinds of statements:

1. exact mathematical identities;
2. numerical patterns or metaphors;
3. claims about the physical world.

This revision keeps only what can stand on its own. The clean core is not obtained by forcing every thread into one rope. It is obtained by subtraction: keep the geometry, keep the number theory, and mark everything else as historical or speculative unless it carries its own independent evidence.

The new spine. The revised Volume I has two exact engines and one exact address layer:

- the **downward cone cascade** ($\downarrow$), governed by $\sqrt{2}$ in the 45° cone;
- the **inward concentric operator** ($\odot$), governed by $\sqrt{3}$ through sphere–cube recursion;
- the **divisor-band address law**, where rational numbers may recover exact grid addresses and, through a separate convention, text.

The first two are geometric. The third is number-theoretic. None of them proves a theory of physical constants. The two geometric operators are shown together in Figure 1.

Standing rule. A named constant must be tagged at first mention as *derived*, *measured*, *posited*, or *imported*. A visual analogy must not be presented as a proof. A recovered address must not be mistaken for recovered meaning.

2 The 45° cone as an exact laboratory

Let a right circular cone have height H, base radius R, and half-angle α so that $\tan\alpha = R/H$. A sphere centered on the cone axis at apex-distance z is tangent to the cone wall exactly when

$$r = z \sin\alpha.$$

The distinguished case is the 45° cone, where $H = R$ and $\alpha = 45^\circ$. We normalize with $H = R = \pi$ because the original Volume I used the π-scaled cone. In this normalization the primary tangent sphere has radius

$$r_0 = \frac{\pi}{\sqrt{2}}.$$

In the old language this value was promoted as a governing physical scalar. In this revision it is treated simply as what it is in this geometry: the radius of the primary tangent sphere in a 45° cone of height and radius π.

The figure is schematic, but it separates two mechanisms correctly. The cone is drawn with its apex downward so the silver-cascade direction matches the actual trapped-tip geometry. In the full 3D geometry, the center of S_0 lies on the cone base plane, so the sphere's equatorial plane coincides with that base plane: one hemisphere lies inside the finite cone and the other lies outside the cone base. The 2D sketch should be read with that alignment in mind, even if the perspective is only schematic. The downward operator generates an *infinite* trapped-tip cascade, illustrated here by a short finite prefix of shrinking spheres; the full chain continues asymptotically to the cone apex. The inward operator contracts nested shapes at a fixed center. Their constants differ for a reason: one is a 45° cone-wall tangency problem, the other is an inscribed cube problem.

3 The downward operator: axial fill and rational void

Place an infinite sequence of spheres $\{S_n\}_{n\geq 1}$ on the cone axis, each tangent to the wall, tangent to its successor, and with S_1 tangent to the base plane. If z_n is the center height from the apex and r_n is the radius, then

$$r_n = z_n \sin\alpha.$$

Successive tangency gives

$$z_n - z_{n+1} = r_n + r_{n+1},$$

so the radii form a geometric progression

$$\frac{r_{n+1}}{r_n} = q = \frac{1-\sin\alpha}{1+\sin\alpha}.$$

Theorem 3.1 (Axial chain fill fraction)**.** *For the infinite axial tangent chain in a right circular cone,*

$$\frac{V_{\text{occ}}}{V_{\text{cone}}} = \frac{2H^2}{3H^2+4R^2} = \frac{2}{3+4\tan^2\alpha} = \frac{2\cos^2\alpha}{3+\sin^2\alpha}.$$

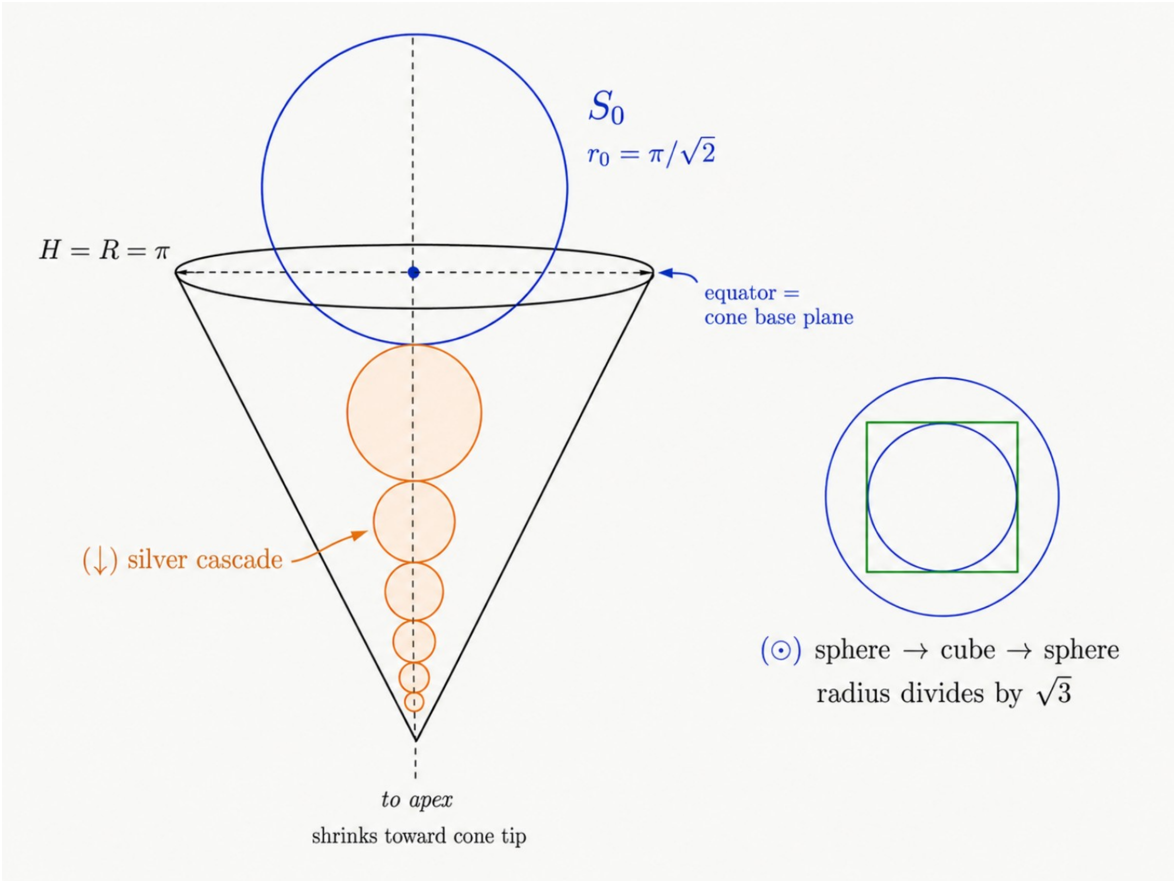


Figure 1: The two independent exact operators of the 45° cone (apex downward): the downward silver cascade (↓), governed by $\sqrt{2}$, with primary tangent sphere S_0 of radius $r_0 = \pi/\sqrt{2}$; and the inward concentric operator (⊙), a sphere–cube–sphere recursion whose radius divides by $\sqrt{3}$. See the surrounding text for the full 3D reading.

Proof. The first radius is $r_1 = H \sin\alpha/(1 + \sin\alpha)$. Therefore

$$V_{\text{occ}} = \sum_{n=1}^{\infty} \frac{4}{3}\pi r_n^3 = \frac{4}{3}\pi \frac{r_1^3}{1 - q^3}.$$

Using

$$1 - q^3 = 1 - \left(\frac{1 - \sin\alpha}{1 + \sin\alpha}\right)^3 = \frac{2\sin\alpha(3 + \sin^2\alpha)}{(1 + \sin\alpha)^3},$$

and $V_{\text{cone}} = \frac{1}{3}\pi R^2 H = \frac{1}{3}\pi H^3 \tan^2\alpha$, the stated ratios follow by simplification. □

Corollary 3.2 (45° cone)**.** *If $H = R$, so $\alpha = 45°$, then*

$$\frac{V_{\text{occ}}}{V_{\text{cone}}} = \frac{2}{7}, \qquad \frac{V_{\text{empty}}}{V_{\text{cone}}} = \frac{5}{7}.$$

Now specialize to the normalized 45° cone with $H = R = \pi$. The primary sphere S_0 has radius $r_0 = \pi/\sqrt{2}$ and center height $z = \pi$. Let T be the trapped tip region beneath S_0, from $z = 0$ to $z = \pi/2$, outside the lower spherical cap.

Lemma 3.3 (Trapped tip volume)**.**

$$V_{\text{trap}} = \frac{3 - 2\sqrt{2}}{12}\pi^4.$$

Proof. The cone volume up to $z = \pi/2$ is $\pi^4/24$. The corresponding lower cap of S_0 has volume $\frac{\pi^4}{24}(4\sqrt{2} - 5)$. Subtracting gives

$$V_{\text{trap}} = \frac{\pi^4}{24}(6 - 4\sqrt{2}) = \frac{3 - 2\sqrt{2}}{12}\pi^4.$$

□

Inside T, place the trapped cascade of tangent spheres. The contraction factor is

$$\lambda = \frac{1 - \sin 45^\circ}{1 + \sin 45^\circ} = (\sqrt{2} - 1)^2 = 3 - 2\sqrt{2} \approx 0.1716.$$

With first trapped radius $R_1 = \lambda r_0$ and $R_n = R_1 \lambda^{n-1}$, the cascade volume is

$$V_{\text{cascade}} = \frac{4}{3}\pi \frac{R_1^3}{1 - \lambda^3} = \frac{10 - 7\sqrt{2}}{42}\pi^4.$$

Theorem 3.4 (Rational void)**.** *The remaining void after the trapped cascade is packed into T is*

$$V_{\text{void}} = V_{\text{trap}} - V_{\text{cascade}} = \frac{\pi^4}{84},$$

so, since $V_{\text{cone}} = \pi^4/3$,

$$\frac{V_{\text{void}}}{V_{\text{cone}}} = \frac{1}{28}.$$

Proof. Over the common denominator 84,

$$\frac{3 - 2\sqrt{2}}{12} - \frac{10 - 7\sqrt{2}}{42} = \frac{21 - 14\sqrt{2}}{84} - \frac{20 - 14\sqrt{2}}{84} = \frac{1}{84}.$$

The irrational terms cancel exactly. □

The important feature is not only that the result is small. It is that an irrational silver-ratio cascade leaves a rational hole.

4 Curvature ladder: Pell structure in $\mathbb{Z}[\sqrt{2}]$

In bend coordinates $b_i = 1/r_i$, five mutually tangent spheres satisfy the Soddy–Gossett relation

$$\left(\sum_{i=1}^{5} b_i\right)^2 = 3\sum_{i=1}^{5} b_i^2.$$

For the axial cascade in the 45° cone, the contraction factor is $\lambda = 3 - 2\sqrt{2}$. Therefore the curvature ratio is

$$\frac{k_{n+1}}{k_n} = \frac{1}{\lambda} = 3 + 2\sqrt{2} = (1 + \sqrt{2})^2.$$

With a convenient normalization,

$$k_n = (1 + \sqrt{2})^{2n-1}.$$

Writing $k_n = a_n + b_n\sqrt{2}$ yields the Pell ladder

$$1+\sqrt{2}, \quad 7+5\sqrt{2}, \quad 41+29\sqrt{2}, \quad \dots$$

with integer pairs $(1,1),(7,5),(41,29),\dots$ and $a_n/b_n \to \sqrt{2}$.

This is the clean arithmetic content behind the phrase "silver gears." The nickname is optional. The result is not.

5 Macroscopic closure and the bicone

The same π normalization produces a coarse-scale area correspondence. In the 45° cone with $H = R = \pi$ and $r_0 = \pi/\sqrt{2}$,

$$\pi R^2 = \pi^3,$$

and the upper hemisphere of the primary sphere has area

$$2\pi r_0^2 = 2\pi \left(\frac{\pi}{\sqrt{2}}\right)^2 = \pi^3.$$

The pole-to-equator chord has length $r_0\sqrt{2} = \pi$.

If a symmetric bicone B is formed by adjoining a congruent inverted cone, then

$$V(B) = 2 \cdot \frac{1}{3}\pi^4 = \frac{2}{3}\pi^4,$$

whereas

$$V(S_0) = \frac{4}{3}\pi \left(\frac{\pi}{\sqrt{2}}\right)^3 = \frac{\sqrt{2}}{3}\pi^4.$$

Therefore

$$\frac{V(S_0)}{V(B)} = \frac{1}{\sqrt{2}}.$$

This section is a consistency check, not a new physical law. It confirms that the π-scaled cone and its primary tangent sphere have a tidy macroscopic closure.

6 The inward concentric operator

The inward operator is independent of the downward cone cascade. Starting with a sphere S_n of radius r_n, inscribe a cube C_n in it. Then inscribe the next sphere S_{n+1} in that cube. This defines

$$S_n \to C_n \to S_{n+1}.$$

For a cube inscribed in a sphere, the cube's space diagonal equals the sphere diameter:

$$s_n\sqrt{3} = 2r_n,$$

so $s_n = 2r_n/\sqrt{3}$. The next inscribed sphere has radius $r_{n+1} = s_n/2$, hence

$$r_{n+1} = \frac{r_n}{\sqrt{3}}.$$

The volume and area ratios are classical inscribed-solid ratios, assembled here as an operator:

$$\frac{V(C_n)}{V(S_n)} = \frac{2}{\pi\sqrt{3}}, \qquad \frac{V(S_{n+1})}{V(C_n)} = \frac{\pi}{6},$$

and

$$\frac{A(C_n)}{A(S_n)} = \frac{2}{\pi}.$$

Over one full sphere–cube–sphere cycle,

$$\frac{2}{\pi\sqrt{3}} \cdot \frac{\pi}{6} = \frac{1}{3\sqrt{3}},$$

so the transcendental factor π cancels.

7 The $L^1 \supset L^2 \supset L^\infty$ metric bridge

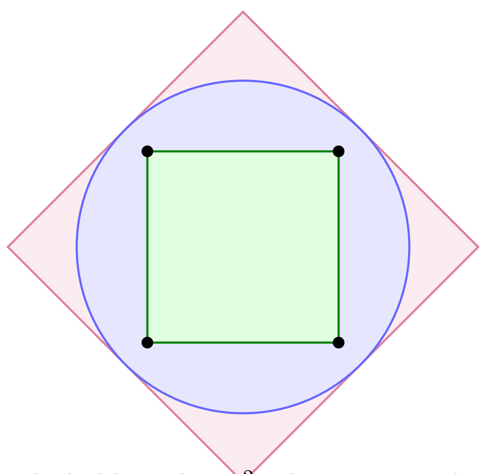
Let S_0 be the Euclidean ball in $\mathbb{R}^3$ of radius $r_0 = \pi/\sqrt{2}$:

$$S_0 = \{(x, y, z) : x^2 + y^2 + z^2 \leq \pi^2/2\}.$$

Then S_0 sits between an outer L^1 ball and an inner L^∞ ball:

$$\{(x, y, z) : |x| + |y| + |z| \leq \pi\sqrt{3/2}\} \supset S_0 \supset \{(x, y, z) : \max(|x|, |y|, |z|) \leq \pi/\sqrt{6}\}.$$

For the inner cube, a vertex is $(\pm a, \pm a, \pm a)$, so tangency to S_0 requires $\sqrt{3}a = \pi/\sqrt{2}$ and $a = \pi/\sqrt{6}$. For the outer octahedron, the face $x+y+z = b$ has distance $b/\sqrt{3}$ from the origin, so tangency requires $b/\sqrt{3} = \pi/\sqrt{2}$ and $b = \pi\sqrt{3/2}$.



L^1 octahedral bound $\supset L^2$ sphere $\supset L^\infty$ cubic core

The eight points $(\pm\pi/\sqrt{6}, \pm\pi/\sqrt{6}, \pm\pi/\sqrt{6})$ are the vertices of the inscribed cube. This is an exact eight-point directional skeleton. It does not by itself prove any rotational, acoustic, or entropy claim.

8 Addressable structure: from fraction to address

The companion theorem-addressed message paper uses a different but philosophically aligned exact structure. Instead of packing spheres, it packs rational numbers into a triangular fractional grid.

For integers $n \geq 1$ and $1 \leq k \leq n$, define

$$a(n, k) = n - 1 + \frac{k}{n} = \frac{n^2 - n + k}{n}.$$

Chamber n consists of the n values $a(n,1),\dots,a(n,n)$ in $(n-1,n]$.

Let r/s be a positive rational in lowest terms and set

$$n = \lceil r/s \rceil .$$

Then r/s occurs in the grid if and only if

$$s \mid n.$$

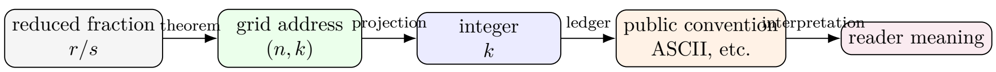
When it occurs, the address is unique, in chamber n, at step

$$k = \frac{n}{s}\bigl(r - s(n-1)\bigr).$$

The band remainder

$$t = r - s(n-1)$$

is not generally the step. The actual step is $k = (n/s)t$. Missing that scale factor breaks the decode.



The connection to the cone paper is not that a cone encodes text. The connection is methodological: exact structure is separated from meaning. In the cone, a volume identity is not a cosmology. In the grid, an address is not a letter until a public convention is supplied.

9 What is no longer a standing claim

The revision does not erase the history of the project. It reclassifies it. The following table states the firewall.

Earlier Volume I claim family	**Revision status**	**Reason**
$\pi/\sqrt{2}$ as a universal governing scalar	Replaced by geometric provenance	In the 45° cone, $\pi/\sqrt{2}$ is the primary tangent sphere radius under the $H = R = \pi$ normalization. That is exact, but it does not govern physics by itself.
Speed of light expressions such as $(\sqrt{2}\pi)^{10}$	Historical only, not standing	The expression is dimensionless and unit-dependent comparisons to c cannot derive a physical constant.
Cyclic or repeating decimal readings of $\pi/\sqrt{2}$	Rejected as law	$\pi/\sqrt{2}$ is irrational. Finite decimal approximations are approximations, not hidden periodicity.
Atomic, superconducting, Schumann, and market resonance claims	Removed from core	These require domain-specific mechanisms and out-of-sample testing. The cone identities do not supply that evidence.

Near-7/8 acoustic or entropy interpretations	Quarantined as heuristic if retained elsewhere	The $L^1/L^2/L^\infty$ bridge gives an exact eight-point skeleton, but no causal dynamical law follows from that alone.

This is not a demotion of the exact results. It is protection for them. The mathematical core becomes stronger when it stops carrying claims it cannot certify.

10 Summary: the honest Volume I

The revised Volume I says less and therefore says it better.

- The 45° cone has an exact axial fill fraction: 2/7 occupied and 5/7 empty.
- Its trapped-tip silver cascade leaves an exact rational void: $V_{\text{void}} = \pi^4/84$, i.e. 1/28 of the cone volume.
- Its curvature ladder lives naturally in $\mathbb{Z}[\sqrt{2}]$ and follows Pell recursions.
- Its inward concentric operator contracts radii by $1/\sqrt{3}$ and preserves the ratios $2/(\pi\sqrt{3})$, $\pi/6$, and $2/\pi$.
- Its primary sphere mediates an exact $L^1 \supset L^2 \supset L^\infty$ bridge.
- The address grid recovers numbers by a divisibility law, while public conventions and readers supply text and meaning.

The shared discipline is simple:

> Declare the ledger before projecting. Verify the identity before naming it. Let structure be exact without forcing it to certify meaning.

This is the lead companion paper because it teaches the rule that now governs the whole project: exactness first, provenance always, interpretation last.

A Verification witness

The accompanying packet includes a standalone Python/SymPy verification script. Its purpose is modest: reproduce the headline symbolic identities exactly. The script checks the axial fill reduction, the 45° specialization, the trapped-tip volumes, the rational void cancellation, the Pell powers, the inward operator ratios, and the metric bridge constants.

A verification script is not a substitute for proof, but it is a useful independent witness. It catches algebra slips, scale mistakes, and accidental numerical overclaims.

B Human-readable provenance notes

- This document is a major revision of *Volume I: Scalar Emergence and Symbolic Foundations* (April 2025).
- The cone identities are consolidated from the salvaged 45° cone geometry paper.
- The address layer is included because it is the companion exact-number structure that emerged from the same divisibility instincts.
- The physical scalar thesis is retained only as historical background unless separately supported by domain evidence.

References

[1] J. Huckstead, *Volume I: Scalar Emergence and Symbolic Foundations*, April 2025 historical manuscript.

[2] J. Huckstead, *Tangent-Sphere Cascades in the* 45° *Cone: Exact Volume, Curvature, and Metric Identities*, 2026 revision notes.

[3] J. Huckstead, *A Message Hidden as an Address: Where a Theorem Recovers a Number, and Only You Recover the Letter*, June 2026 capstone manuscript.

[4] F. Soddy, "The Kiss Precise," *Nature* 137, 1021, 1936.

[5] Archimedes, *On the Sphere and Cylinder*, trans. T. L. Heath, 1912 edition.