Triangular Addressing, Null Coordinates, and Reciprocal-Metallic Locks in a Scale-Covariant Plane Geometry

Jeffery Huckstead
Cerebral Graphix

Published Working Draft III
19 July 2026
doi:10.5281/zenodo.21444929
Preceding published version: 10.5281/zenodo.21435545

Abstract

We study a plane-geometric construction generated by the flattened triangular rank $\rho(n,k) = n(n-1)/2+k$, renamed $m = \rho(n,k)$ in the present geometry, together with a cell scale $q > 0$ and a shape parameter $0 < c < 1$. The coordinates $t = qm$ and $s = ct$ determine a rectangle of area $d = st$, a reciprocal hyperbola $xy = d$, and, under a half-normalized null-coordinate transform, a point on $x^2 - y^2 = d$. A second intersection with the same split hyperbola generates a centered rectangle that is similar to the original rectangle for every c. We prove that $c = \sqrt{5} - 2$ is the unique full Euclidean congruence lock and that the same root simultaneously produces a null-coordinate collision and a rigid tangent-wing dissection. The novelty claimed is this forced co-occurrence inside one generator, not the classical coordinate transform, metallic constant, tangent theorem, or continued-fraction machinery separately. The three native collision values are the reciprocals of the metallic means with indices $a = 1, 2, 4$. Their rational cell-grid approximants satisfy generalized Pell equations; for the congruence family this records an exact alternating area-error formula. The triangular address is inherited from the revised Triangular-Fractional Grid foundation, while the divisor-band and totient-shell theorems of that parent are not imported into this geometric theorem line. We also separate address scaling from fixed-extent resolution and state explicit methodological fences for external targets, spectral operators, information measures, and physical interpretations.

Keywords. Null coordinates; pseudo-Euclidean geometry; reciprocal hyperbola; metallic means; continued fractions; Pell equations; triangular numbers.

Status and scope. This is a self-contained published working draft archived at version-specific Zenodo record 21444929; the preceding published version is record 21435545. The results claimed as theorems below are internal identities of the construction. References to rapidity, light-cone coordinates, entropy, spectra, or physics describe exact mathematical structures or clearly labeled research directions; they are not claims that the present construction is a physical law or a generator of an externally supplied constant. This archival source remains the authority for theorem wording; companion and website material may summarize but not extend it.

1 Introduction

The construction begins with the equal-area relation

$$ts = d, \qquad s = \frac{d}{t}, \tag{1.1}$$

so equal-area rectangles lie on the reciprocal hyperbola $xy = d$. The horizontal coordinate is attached to a triangular chamber address and the vertical coordinate is controlled by an aspect parameter c. This simple starting point produces three linked structures:

(i) a reciprocal rectangle on $xy = d$;
(ii) a half-sum/half-difference image on $x^2 - y^2 = d$;
(iii) a centered rectangle cut from the rail $y = cx$ and the split hyperbola.

The central result is not an isolated numerical coincidence. The original and centered rectangles form a universal similarity family, and the positive root of

$$c^2 + 4c - 1 = 0 \tag{1.2}$$

is the unique point at which the similarity becomes full Euclidean congruence. The same value controls a collision on $x^2 - y^2 = d$ and a piecewise rigid dissection into tangent wings.

The arithmetic address lattice supplies a second exact layer. Rational heights $c = r/m$ give rectangular arrays of q-cells, whereas the irrational collision locks have continued fractions with period one after their zero integer part. Their reciprocals are strictly purely periodic metallic means. The convergents solve generalized Pell equations, yielding an exact bridge between finite cell grids and the irrational geometric locks. Classical ingredients are cited below; the proved-here novelty seat is the bundle forced by one generator and the native selection of indices $\{1, 2, 4\}$, stated with the conservative boundary that no exact prior appearance was located in the bounded search.

2 Triangular address and cell geometry

Definition 2.1 (Address generator)**.** Let

$$n \geq 1, \qquad 1 \leq k \leq n, \qquad \rho(n,k) = \frac{n(n-1)}{2} + k, \qquad m := \rho(n,k). \tag{2.1}$$

For $q > 0$ and $0 < c < 1$, define

$$t = qm, \qquad s = ct, \qquad d = st = ct^2. \tag{2.2}$$

The original rectangle is

$$R = [0,t] \times [0,s], \qquad A_R = ts = d. \tag{2.3}$$

The flattened rank ρ is the triangular chamber-step address proved in the revised Triangular-Fractional Grid foundation [1]. The present paper renames that integer m and forms the geometric length $t = qm$. This is an inheritance of indexing only: the TFG divisor-band and totient-shell theorems depend on reduction of its separate rational projection and are not asserted here. Triangular flattening itself belongs to the classical Cantor-pairing lineage [2].

The internal divider list is

$$J = [1, \ldots, m-1]. \tag{2.4}$$

Together with $x = 0$ and $x = t$, the divider positions

$$x_j = qj, \qquad j = 0, 1, \ldots, m, \tag{2.5}$$

produce exactly m cells of width q.

Proposition 2.2 (Inverse address)**.** *The map* $(n, k) \mapsto m$ *in* (2.1) *is a bijection from* $\{(n, k) : n \geq 1,\ 1 \leq k \leq n\}$ *to the positive integers. Its inverse is*

$$n = \left\lceil \frac{\sqrt{8m+1} - 1}{2} \right\rceil, \qquad k = m - \frac{n(n-1)}{2}. \tag{2.6}$$

Proof. The n-th chamber is the integer interval

$$T_{n-1} < m \leq T_n, \qquad T_n = \frac{n(n+1)}{2}.$$

Solving $m \leq T_n$ for the least admissible n gives (2.6); subtracting T_{n-1} gives k. □

Thus chamber n contains n consecutive q-cells and has width nq. The fine-grid increment is always q; nq is a block width, not a second fundamental lattice step.

Proposition 2.3 (q-dilation covariance)**.** *For fixed* (m, c)*, the normalized coordinates*

$$\bar{x} = \frac{x}{q}, \qquad \bar{y} = \frac{y}{q} \tag{2.7}$$

remove q *from every dimensionless geometric relation.*

Proof. Equations (2.2) give

$$(\bar{t}, \bar{s}) = (m, cm), \qquad \frac{d}{q^2} = cm^2.$$

Consequently

$$xy = d \iff \bar{x}\bar{y} = cm^2, \qquad x^2 - y^2 = d \iff \bar{x}^2 - \bar{y}^2 = cm^2,$$

and $y = cx \iff \bar{y} = c\bar{x}$. The divider lattice becomes $\bar{x} = J$. Therefore lengths scale as q, areas as q^2, and dimensionless ratios are q-independent. □

Remark 2.4 (Two control laws)**.** In *address mode*, m is fixed and $t = qm$; changing q dilates the cell width and total extent together. In a separate *resolution mode*, a macroscopic extent L is fixed and $q = L/m$; increasing m then refines a fixed interval. Conclusions from these modes must not be mixed.

3 Null coordinates and the projective involution

We use the standard half-sum/half-difference, or null-coordinate, change of variables familiar from radar coordinates and the hyperbolic-number plane [3, 4, 5]. The coordinate change is classical; the role it plays inside the present generator is the object under study.

Define

$$u = \frac{t+s}{2}, \qquad w = \frac{t-s}{2}, \qquad U = (u, w). \tag{3.1}$$

The inverse relations are

$$t = u + w, \qquad s = u - w. \tag{3.2}$$

Proposition 3.1 (Split-hyperbola image)**.** *The point U lies on*

$$H_2 : \quad x^2 - y^2 = d. \tag{3.3}$$

Proof. Directly,

$$u^2 - w^2 = \frac{(t+s)^2 - (t-s)^2}{4} = ts = d.$$

□

In matrix form,

$$\begin{pmatrix} u \\ w \end{pmatrix} = A \begin{pmatrix} t \\ s \end{pmatrix}, \qquad A = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{3.4}$$

Here

$$\det A = -\frac{1}{2}, \qquad A^2 = \frac{1}{2} I. \tag{3.5}$$

Writing $A = Q/\sqrt{2}$ makes Q orthogonal with $\det Q = -1$. Thus the Euclidean action is a reflection followed by scaling $1/\sqrt{2}$, not a rotation.

The slope of U is

$$r_U = \frac{w}{u} = \frac{1-c}{1+c}. \tag{3.6}$$

Define

$$f(c) = \frac{1-c}{1+c}. \tag{3.7}$$

Equation (3.6) shows that f is the projective action of the null-coordinate transform on slopes. Since A^2 is a scalar multiple of the identity,

$$\boxed{f(f(c)) = c.} \tag{3.8}$$

The positive branch of H_2 admits the exact parameterization

$$U = \sqrt{d}\,(\cosh\phi, \sinh\phi), \qquad \phi = -\frac{1}{2}\log c. \tag{3.9}$$

This ϕ is a mathematical hyperbolic angle, or rapidity coordinate, in the standard sense [6, 3]. At fixed t, however, varying c changes $d = ct^2$; the native c-slider crosses norm shells rather than moving along one fixed Lorentz orbit. The separate normalization

$$U^{\#} = \frac{U}{\sqrt{c}} = t(\cosh\phi, \sinh\phi), \qquad (u^{\#})^2 - (w^{\#})^2 = t^2, \tag{3.10}$$

does lie on a fixed shell.

4 Fixed and rail intersections

Let

$$H_1 : \quad xy = d \tag{4.1}$$

be the reciprocal hyperbola, and let

$$g = \frac{1+\sqrt{5}}{2}. \tag{4.2}$$

Proposition 4.1 (Fixed positive intersection)**.** *The positive intersection of H_1 and H_2 is*

$$P = \left(\sqrt{dg}, \sqrt{\frac{d}{g}}\right). \tag{4.3}$$

It satisfies

$$OP^2 = \sqrt{5}\, d. \tag{4.4}$$

Proof. The identities

$$g - \frac{1}{g} = 1, \qquad g + \frac{1}{g} = \sqrt{5}$$

give $P_x P_y = d$, $P_x^2 - P_y^2 = d$, and $P_x^2 + P_y^2 = d(g + g^{-1}) = \sqrt{5}\, d$. □

The positive intersection of the rail $y = cx$ with H_2 is

$$I_+ = (X, Y), \qquad X = \sqrt{\frac{d}{1-c^2}}, \qquad Y = cX. \tag{4.5}$$

The centered blue rectangle is

$$\mathcal{I} = [-X, X] \times [-Y, Y]. \tag{4.6}$$

Its area and aspect ratio are

$$A_{\mathcal{I}} = 4XY = \frac{4dc}{1-c^2}, \qquad \frac{Y}{X} = c. \tag{4.7}$$

5 Universal similarity and the congruence lock

Although the side-ratio calculation is elementary, the following theorem is the organizing consolidation of the construction: one similarity factor governs side, area, perimeter, and diagonal across the full one-parameter family.

Theorem 5.1 (Universal similarity)**.** *For every $0 < c < 1$, the rectangles R and $\mathcal{I}$ are similar. The linear factor from R to $\mathcal{I}$ is*

$$\Sigma_{\mathcal{I}/R} = \frac{2X}{t} = \frac{2Y}{s} = 2\sqrt{\frac{c}{1-c^2}}. \tag{5.1}$$

Consequently,

$$\frac{A_{\mathcal{I}}}{A_R} = \Sigma_{\mathcal{I}/R}^2 = \frac{4c}{1-c^2}. \tag{5.2}$$

Proof. Using $d = ct^2$ in (4.5),

$$\frac{X}{t} = \sqrt{\frac{c}{1-c^2}}.$$

Since $Y = cX$ and $s = ct$, the same ratio equals Y/s. The side ratios therefore agree, proving similarity and (5.1). Squaring the linear factor gives (5.2). □

Theorem 5.2 (Full Euclidean congruence lock)**.** *The following conditions are equivalent:*

(a) $\mathcal{I} \cong R$;
(b) $A_{\mathcal{I}} = A_R$;
(c) the perimeters of $\mathcal{I}$ and R are equal;

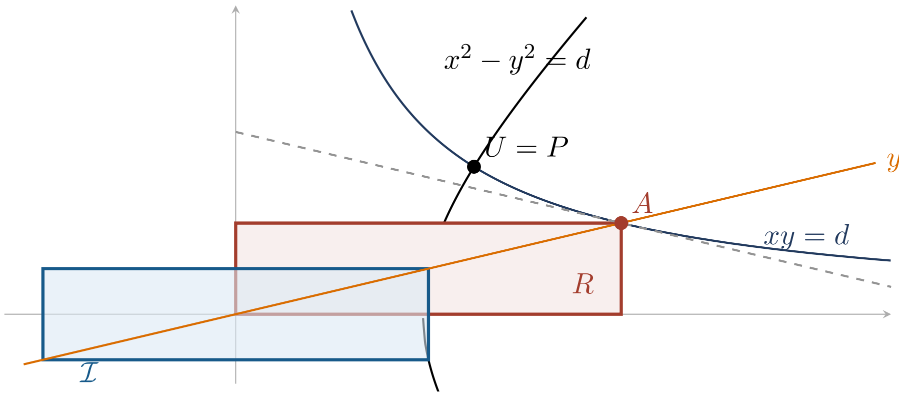

Figure 1: Normalized schematic at the congruence lock $c = \sqrt{5} - 2$, with $t = 1$. The centered blue rectangle $\mathcal{I}$ is congruent to R, while the null-coordinate point U coincides with the fixed intersection P.

(d) the diagonals of $\mathcal{I}$ and R are equal;
(e) $2X = t$ and $2Y = s$.

They hold at the unique value

$$\boxed{c_{\mathrm{B}} = \sqrt{5} - 2 = g^{-3}.} \tag{5.3}$$

Proof. By universal similarity, each condition is equivalent to $\Sigma_{\mathcal{I}/R} = 1$. Equation (5.1) then gives

$$4c = 1 - c^2,$$

or $c^2 + 4c - 1 = 0$. The only root in $(0, 1)$ is $\sqrt{5} - 2$. □

At the lock,

$$\mathcal{I} = \left[-\frac{t}{2}, \frac{t}{2}\right] \times \left[-\frac{s}{2}, \frac{s}{2}\right], \tag{5.4}$$

and translation by $(t/2, s/2)$ maps $\mathcal{I}$ onto R. In their original plotted positions the overlap is only $d/4$; congruence does not mean coincidence.

6 Tangent wings and the blue dissection

The intercept and constant-area properties of tangents to a rectangular hyperbola are classical conic geometry [10]. At $A = (t, s)$, the reciprocal hyperbola $y = d/x$ has tangent slope

$$-\frac{d}{t^2} = -\frac{s}{t} = -c. \tag{6.1}$$

Hence its tangent line is

$$T_A : \quad y = 2s - cx. \tag{6.2}$$

Its intercepts are $(0, 2s)$ and $(2t, 0)$. The two wings outside R are

$$W_L = \operatorname{conv}\{(0, s), (0, 2s), (t, s)\}, \tag{6.3}$$
$$W_R = \operatorname{conv}\{(t, 0), (t, s), (2t, 0)\}. \tag{6.4}$$

Each has area $d/2$.

The descending diagonal of $\mathcal{I}$ is $y = -cx$, parallel to T_A. It cuts $\mathcal{I}$ into

$$B_- = \operatorname{conv}\{(-X,-Y),(-X,Y),(X,-Y)\}, \tag{6.5}$$
$$B_+ = \operatorname{conv}\{(-X,Y),(X,Y),(X,-Y)\}. \tag{6.6}$$

Each blue half has legs $2X, 2Y$, whereas each tangent wing has legs t, s. They are therefore similar for every c.

Corollary 6.1 (Rigid dissection at the balance lock)**.** *At $c = \sqrt{5}-2$, the blue halves are congruent to the tangent wings. Explicit isometries are*

$$F_-(x,y) = \left(x + \frac{t}{2},\, y + \frac{3s}{2}\right), \tag{6.7}$$
$$F_+(x,y) = \left(-x + \frac{3t}{2},\, -y + \frac{s}{2}\right), \tag{6.8}$$

with $F_-(B_-) = W_L$ and $F_+(B_+) = W_R$.

The first map is a translation and the second is a translated half-turn. Thus the dissection is one natural piecewise Euclidean action, not one global rigid motion of $\mathcal{I}$.

7 Collision relay and reciprocal metallic means

The three distinguished positive slopes on H_2 are

$$r_U = f(c) = \frac{1-c}{1+c}, \qquad r_I = c, \qquad r_P = \frac{1}{g}. \tag{7.1}$$

Equality of these slopes gives the pairwise collisions in Table 1.

Table 1: Native collisions on H_2.

Collision	Condition	Lock	Quadratic
$I_+ = P$	$c = g^{-1}$	$\frac{1}{g}$	$c^2 + c - 1 = 0$
$U = I_+$	$f(c) = c$	$\sqrt{2}-1$	$c^2 + 2c - 1 = 0$
$U = P$	$f(c) = g^{-1}$	$\sqrt{5}-2$	$c^2 + 4c - 1 = 0$

At $c = g^{-1}$, one also has $I_+ = P = A$. At $c = \sqrt{5}-2$, the collision $U = P$ coincides with the full rectangle-congruence lock.

Definition 7.1 (Reciprocal metallic family)**.** For a positive integer a, define

$$\mu_a = \frac{a + \sqrt{a^2+4}}{2}, \qquad c_a = \mu_a^{-1} = \frac{\sqrt{a^2+4}-a}{2}. \tag{7.2}$$

This is the classical metallic-means family [7]. It satisfies

$$c_a^2 + ac_a - 1 = 0, \qquad c_a = [0; \overline{a}], \qquad \mu_a = [\overline{a}]. \tag{7.3}$$

Thus c_a is periodic with period one immediately after its zero integer part, while μ_a is strictly purely periodic. For $a = 4$,

$$\mu_4 = 2 + \sqrt{5} = g^3, \qquad c_4 = \sqrt{5} - 2 = g^{-3}, \tag{7.4}$$

a known identity recorded in the standard sequence literature [11]. The present result exploits this value; it does not claim the identity itself. The native collision locks are the members with indices

$$a = 1,\ 2,\ 4. \tag{7.5}$$

The $a = 3$ reciprocal metallic mean is not a native collision lock of this construction.

8 The OUP triangle

Let $O = (0, 0)$ and set

$$\phi = -\frac{1}{2}\log c, \qquad \alpha = \frac{3}{2}\log g. \tag{8.1}$$

Then

$$U = \sqrt{d}(\cosh\phi, \sinh\phi), \qquad P = \sqrt{d}(\cosh\alpha, \sinh\alpha). \tag{8.2}$$

Define

$$M = uP_x - wP_y, \qquad D = uP_y - wP_x. \tag{8.3}$$

Proposition 8.1 (Minkowski Gram identity)**.** *The quantities in* (8.3) *satisfy*

$$M = d\cosh(\phi - \alpha), \qquad D = d\sinh(\alpha - \phi), \tag{8.4}$$

and therefore

$$\boxed{M^2 - D^2 = d^2.} \tag{8.5}$$

Moreover,

$$\frac{[\triangle OUP]}{d} = \frac{1}{2}\left|\sinh(\alpha - \phi)\right| = \frac{1}{4}\left|\sqrt{g^3 c} - \frac{1}{\sqrt{g^3 c}}\right|. \tag{8.6}$$

Proof. The first two formulas are the hyperbolic addition identities applied to (8.2). Their difference of squares gives (8.5). Since D is the determinant of the columns U and P, the Euclidean triangle area is $|D|/2$, yielding (8.6). □

The triangle degenerates uniquely at $c = g^{-3} = \sqrt{5} - 2$. The separate native equation $[\triangle OUP] = d$ has the unique solution in $(0, 1)$

$$c = g^{-9} = (\sqrt{5} - 2)^3 = 17\sqrt{5} - 38. \tag{8.7}$$

No physical or spectral meaning is asserted for this additional area lock.

9 Rational cell locks and Pell-optimal approximation

If

$$c = \frac{r}{m}, \qquad 1 \le r < m, \tag{9.1}$$

then

$$t = mq, \qquad s = rq, \qquad d = mrq^2. \tag{9.2}$$

Adding horizontal dividers at multiples of q produces an exact $m \times r$ array of q-squares. The square-cell lock is the special case $r = 1$, or $c = 1/m$.

Theorem 9.1 (Generalized Pell bridge)**.** *Let r_j/m_j be the convergents of*

$$c_a = [0; \overline{a}].$$

Then the numerator and denominator sequences satisfy

$$r_{j+1} = ar_j + r_{j-1}, \qquad m_{j+1} = am_j + m_{j-1}, \tag{9.3}$$

and

$$\boxed{m_j^2 - am_jr_j - r_j^2 = (-1)^j} \tag{9.4}$$

when the first convergent $1/a$ is indexed by $j = 1$.

Proof. The recurrence (9.3) is the standard convergent recurrence for $[0; \overline{a}]$, and the determinant identity for consecutive convergents alternates in sign [8, 9]. Equivalently, direct substitution of (9.3) shows that the quadratic form $m^2 - amr - r^2$ changes sign at each step and has initial value -1. □

Corollary 9.2 (Exact balance-area error)**.** *The following formula is recorded here as the geometric consequence of the classical convergent determinant identity. For the congruence family $a = 4$, substitute $c = r/m$ into (5.2). Then*

$$\frac{A_{\mathcal{I}}}{A_R} - 1 = -\frac{m^2 - 4mr - r^2}{m^2 - r^2}. \tag{9.5}$$

At the Pell convergents,

$$\boxed{\frac{A_{\mathcal{I}}}{A_R} - 1 = \mp\frac{1}{m^2 - r^2}.} \tag{9.6}$$

The first three rational approximants illustrate the alternating exact error:

$$\frac{r}{m} = \frac{1}{4}: \quad \frac{A_{\mathcal{I}}}{A_R} = \frac{16}{15}, \tag{9.7}$$

$$\frac{r}{m} = \frac{4}{17}: \quad \frac{A_{\mathcal{I}}}{A_R} = \frac{272}{273}, \tag{9.8}$$

$$\frac{r}{m} = \frac{17}{72}: \quad \frac{A_{\mathcal{I}}}{A_R} = \frac{4896}{4895}. \tag{9.9}$$

Thus the visually prominent square-cell value $c = 1/4$ is the first Pell-optimal rational approximation to the exact balance lock $\sqrt{5} - 2$. No finite rational cell grid realizes an irrational collision lock exactly.

10 Boundary and overlap regimes

At fixed (n, k, q), as $c \to 0^+$,

$$s, d, X, Y \to 0, \qquad \frac{A_{\mathcal{I}}}{A_R} \to 0.$$

As $c \to 1^-$, the rectangle R approaches a finite square, while

$$X, Y, A_{\mathcal{I}} \to \infty, \qquad \frac{Y}{X} \to 1.$$

The rigorous description is an unbounded family of rectangles whose aspect ratio tends to 1; there is no finite rail intersection at $c = 1$.

The positive quarter of $\mathcal{I}$ is $[0, X] \times [0, Y]$, with common scale

$$\frac{X}{t} = \frac{Y}{s} = \sqrt{\frac{c}{1 - c^2}}. \tag{10.1}$$

It lies inside R for $0 < c \le g^{-1}$, equals R at $c = g^{-1}$, and contains R for $g^{-1} < c < 1$.

11 External targets, information, and physics fences

11.1 Classification is not generation

Let an external height $h > 0$ be supplied and impose $s = h$. From $s = cqm$,

$$c = \frac{h}{qm}. \tag{11.1}$$

The condition $c < 1$ is $m > h/q$, so the least admissible address is

$$m_{\min}(h) = \left\lfloor \frac{h}{q} \right\rfloor + 1. \tag{11.2}$$

Every positive target can therefore be assigned an address and fitted c. This is exact classification, but it is not prediction or generation of h.

For the canonical $q = 1/2$, address coordinates belong to the half-grid

$$\left\{ \frac{m}{2} : m \in \mathbb{Z}_{>0} \right\}.$$

An external value strictly between 14 and 14.5, for example, can be a continuous height or intersection but cannot be an address tick. This excludes only the address identification; it does not rule out every conceivable future operator spectrum.

11.2 Information requires a measure

The bijection $(n, k) \leftrightarrow m \leftrightarrow t$ is lossless when q is known. It is an encoding, not Shannon compression. In fixed-extent resolution mode, a declared cell distribution $\{p_j(q)\}$ would define

$$H(q) = -\sum_j p_j(q) \log p_j(q), \qquad D_1 = \lim_{q \to 0} \frac{H(q)}{\log(1/q)}, \tag{11.3}$$

when the limit exists. Nonuniformity alone does not imply $D_1 < 1$: regular absolutely continuous distributions on an interval generally still have information dimension 1. A physical entropy requires an operational ensemble, coarse-graining, dynamics, or density operator in addition to the geometry.

11.3 Physics as guide, not fitted conclusion

The null-coordinate identity and hyperbolic rapidity are exact mathematics shared with Minkowski geometry. They do not, by themselves, make t, s, u, w spacetime or momentum coordinates. A physical claim would require at least:

1. operational meanings and units for the coordinates;
2. a fixed invariant and evolution law;
3. independently specified initial or boundary conditions;
4. a fixed operator or probability law;
5. predictions evaluated on values not used to design or tune the model.

The present result is therefore an exact, scale-covariant pseudo-Euclidean geometric scaffold. Physics may guide its questions without being claimed as its conclusion.

12 Related structures and attribution

The construction deliberately joins several established structures without claiming their individual invention.

(i) The triangular flattening and inverse square-root address belong to the Cantor-pairing lineage, while the specific parent record used here is the revised TFG rank $\rho(n,k)$ [2, 1].

(ii) The half-sum/half-difference transform, split-hyperbola coordinates, hyperbolic-number plane, and rapidity parameter are classical constructions [3, 4, 5, 6].

(iii) The metallic family and its periodic continued fractions are classical [7, 8, 9]; in particular, $\mu_4 = g^3$ is known background [11].

(iv) The tangent-intercept and constant-area facts used in the wing construction are classical conic geometry [10].

Within those cited structures, the proved-here organizing claim is that the single admissible root of $c^2 + 4c - 1 = 0$ forces, in the same generator, the rectangle congruence $\mathcal{I} \cong R$, the collision $U = P$, and the rigid piecewise dissection onto the tangent wings. A bounded prior-art scan found no exact published appearance of that three-event bundle or of the native collision selection $\{1, 2, 4\}$. This is a conservative search report, not a claim of exhaustive priority.

13 Claim and nonclaim ledger

Label	Status	Scope
C1	Classical, cited	Triangular flattening, inverse address, and parent TFG lineage.
C2	Methodological	q-covariance and the separation of address mode from fixed-extent resolution mode.
C3	Classical, cited	Null-coordinate transform and elementary Möbius involution.
C4	Classical, cited	Hyperbolic-angle/rapidity parameter and fixed-shell normalization.
C5	Elementary, proved here	Positive intersection of the two hyperbolas and $OP^2 = \sqrt{5}d$.
C6	Proved here	Universal rectangle similarity as the organizing law for four Euclidean observables.
C7	Proved here; novelty seat	Unique congruence lock and the forced three-event bundle at $c = \sqrt{5} - 2$.
C8	Proved here on classical tangent geometry	Explicit rigid two-piece mapping from the centered rectangle to the tangent wings.

Label	Status	Scope
C9	Proved here; novelty seat	Native collision rule selecting reciprocal-metallic indices $\{1, 2, 4\}$, with $a = 3$ excluded.
C10	Proved here	OUP Gram identity and the separately fenced area-one lock.
C11	Classical machinery; consequence recorded here	Generalized Pell recurrence and the exact alternating geometric area error.
N1	Nonclaim	No physical dynamics, causal law, cosmology, or empirical constant is derived.
N2	Nonclaim	Fitting an external target is classification, not prediction or generation.
N3	Nonclaim	The address map is lossless indexing, not Shannon compression.
N4	Nonclaim	No independent operator spectrum is inferred from the geometric scaffold.
N5	Nonclaim	C-Geometry inherits TFG's triangular address, not its divisor-band or totient-shell theorems.
N6	Nonclaim	The separate wave/accounting paper is not a descendant of this mathematical theorem family.

14 Conclusion

The construction studied here is generated by three inputs—the inherited triangular rank $\rho(n, k) = n(n-1)/2 + k$, renamed $m = \rho(n, k)$, a cell scale $q > 0$, and a shape parameter $0 < c < 1$—and every structure in the main theorem line descends from the equal-area relation $ts = d$ of (1.1). Read through the half-normalized null transform A of (3.4), that identity sends the reciprocal hyperbola $xy = d$ to the split hyperbola $x^2 - y^2 = d$, converts the reciprocal side exchange into the projective involution f of (3.8), and turns the aspect parameter into the hyperbolic rapidity $\phi = -\frac{1}{2} \log c$. No independent numerical target is required: the golden intersection, the reciprocal-metallic family, the Pell recurrences, and the congruence lock arise internally from this chain of relations.

Three results organize the geometry. First, the original and centered rectangles are similar for every c, with the universal factor $\Sigma_{\mathcal{I}/R} = 2\sqrt{c/(1-c^2)}$ of (5.1) governing side, area, perimeter, and diagonal simultaneously. The apparent coincidences among those quantities are one similarity law seen four ways. Second, that similarity becomes full Euclidean congruence at the unique value $c_{\mathrm{B}} = \sqrt{5} - 2 = g^{-3}$, and the same value simultaneously produces the null-coordinate collision $U = P$ and the rigid tangent-wing dissection of (6.8). Three independent-looking events are forced by one root of $c^2 + 4c - 1 = 0$. Third, the native collision locks are the reciprocal metallic means with indices $a = 1, 2, 4$. Their continued fractions are periodic with period one after the zero integer part, while the reciprocal metallic means are strictly purely periodic. Standard Pell recurrences then yield the exact alternating area error $\mp 1/(m^2 - r^2)$ of (9.6). The square-cell value $c = 1/4$ is the first Pell-optimal rational approximant to the irrational balance lock, and no finite cell grid realizes an irrational lock exactly.

What the construction is, stated precisely, is an exact, scale-covariant, pseudo-Euclidean plane geometry with reciprocal-metallic-mean locks and Pell-optimal rational approximants. Its ratios and c-locks are dimensionless; its lengths scale with q and its areas with q^2. The address bijection is lossless encoding, not compression; the null transform is a reflection followed by scaling, not a rotation; and the rapidity coordinate is a genuine hyperbolic angle, not an assertion about velocity or momentum.

What the construction is not is equally part of the result. The parameters q, m, and c supply enough scaling and classification freedom to assign an address and a fitted c to an externally supplied target, as (11.1) shows; recovering such a value is classification, not prediction. Information-theoretic language requires a declared measure and an operational ensemble that the geometry alone does not provide. The vocabulary of null coordinates, rapidity, and Minkowski structure is shared with physics as exact mathematics, but a physical claim would require operational units, a fixed invariant and evolution law, independent boundary data, a fixed operator or probability law, and predictions tested on values not used to tune the model. None of these is asserted here.

The natural continuations are correspondingly bounded. On the arithmetic side, the reciprocal-metallic locks and their Pell approximants invite study where periodic continued fractions, rather than numerical matching, are the operative property. On the operator side, an independently fixed Laplacian, boundary-value problem, or dynamical law on the address lattice could become substantive only if it produces held-out results without retuning. The present paper makes neither leap. It offers instead a closed and internally consistent geometric scaffold whose apparent coincidences have been reduced to one structural chain, whose special values belong to a named number-theoretic family, and whose limits are stated as plainly as its theorems.

A Compact Desmos implementation

Use integer step 1 for n and k, with $1 \le k \le n$. Freeze $q = 1/2$ exactly. Use a practical closed slider range inside $(0, 1)$, for example $0.001 \le c \le 0.999$.

$$\begin{aligned}
m &= \frac{n(n-1)}{2} + k, & q &= \frac{1}{2}, & t &= qm, \\
s &= ct, & d &= st, & J &= [1, \ldots, m-1], \\
u &= \frac{t+s}{2}, & w &= \frac{t-s}{2}, & g &= \frac{1+\sqrt{5}}{2}, \\
X &= \sqrt{\frac{d}{1-c^2}}, & Y &= \sqrt{\frac{dc^2}{1-c^2}}.
\end{aligned}$$

The main plotted expressions are

$$\begin{aligned}
&xy = d, \qquad x^2 - y^2 = d, \qquad y = cx\{0 \le x \le t\}, \\
&x = qJ\{0 \le y < s\}, \\
&A = (t, s), \qquad U = (u, w), \qquad P = \left(\sqrt{dg}, \sqrt{d/g}\right), \\
&R = [(0,0), (t,0), (t,s), (0,s)], \\
&\mathcal{I} = [(X,Y), (X,-Y), (-X,-Y), (-X,Y)].
\end{aligned}$$

If the rail must visibly reach I_+ for all c, replace its upper restriction t with $\max(t, X)$. At $m = 1$, the intended divider list is empty; guard the divider expression with $m > 1$ if the local Desmos implementation renders a descending range.

Version note

This Published Working Draft III is archived at version-specific DOI 21444929 and supersedes the preceding published working-draft record 21435545 in the Zenodo version lineage. The revision links the address variable to the revised TFG parent, adds the parent/descendant boundary, expands classical attribution, clarifies the novelty seat, corrects continued-fraction terminology, and refines the status wording for the Pell-error result. The independent termial–factorial and divisibility-graph observations from the preceding review draft have been moved into a separately labeled supplementary note so the main C-Geometry theorem line remains geometrically focused. The geometric theorem spine is unchanged. No claim depends on external numerical fitting.

References

[1] J. Huckstead, *Divisor-Band Bijections and Totient Shells in the Triangular-Fractional Grid*, preprint, Zenodo, 2026, doi:10.5281/zenodo.21440877.

[2] G. Cantor, "Ein Beitrag zur Mannigfaltigkeitslehre," *Journal für die reine und angewandte Mathematik*, 84 (1878), 242–258.

[3] H. Bondi, *Relativity and Common Sense: A New Approach to Einstein*, Doubleday/Anchor, New York, 1964.

[4] I. M. Yaglom, *A Simple Non-Euclidean Geometry and Its Physical Basis*, translated by A. Shenitzer, Springer-Verlag, New York, 1979.

[5] G. Sobczyk, "The Hyperbolic Number Plane," *The College Mathematics Journal*, 26(4) (1995), 268–280, doi:10.1080/07468342.1995.11973712.

[6] E. F. Taylor and J. A. Wheeler, *Spacetime Physics*, 2nd ed., W. H. Freeman, New York, 1992.

[7] V. W. de Spinadel, "The metallic means family and multifractal spectra," *Nonlinear Analysis: Theory, Methods & Applications*, 36(6) (1999), 721–745.

[8] G. H. Hardy and E. M. Wright, *An Introduction to the Theory of Numbers*, 6th ed., revised by D. R. Heath-Brown and J. H. Silverman, Oxford University Press, 2008.

[9] A. Ya. Khinchin, *Continued Fractions*, Dover Publications, 1997 reprint.

[10] G. Salmon, *A Treatise on Conic Sections*, 6th ed., Longmans, Green, London, 1879.

[11] OEIS Foundation Inc., *The On-Line Encyclopedia of Integer Sequences*, A098317, "Decimal expansion of $\phi^3 = 2 + \sqrt{5}$," accessed 19 July 2026.