Address-Lattice Arithmetic and Spectral Notes

Supplement to *Triangular Addressing, Null Coordinates, and Reciprocal-Metallic Locks in a Scale-Covariant Plane Geometry*

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Supplementary Note
19 July 2026
doi:10.5281/zenodo.21444929

Abstract

This supplementary note preserves two observations removed from the main C-Geometry manuscript so that its geometric theorem line remains focused. The first is a finite same-index coincidence between Knuth's termial and the factorial. The second is a Laplacian multiplicity formula for a finite divisibility-comparability graph. Both observations are independent of the plane-geometric congruence, collision, tangent-wing, and Pell results in the main paper. Their placement here is organizational, not a change in their mathematical status.

1 Scope and separation

The main manuscript uses the inherited triangular rank

$$m = \rho(n, k) = \frac{n(n-1)}{2} + k$$

as an address for a scale-covariant plane geometry. The two notes below use the same finite address integer, but neither is a consequence of the null-coordinate transform, universal rectangle similarity, reciprocal-metallic lock, tangent-wing dissection, or Pell approximation theorem. Shared indexing does not imply shared mechanism.

2 Finite termial–factorial boundary

We adopt Knuth's termial notation [1]

$$N? := T_N := 1 + 2 + \cdots + N = \frac{N(N+1)}{2} \tag{1}$$

to emphasize its additive symmetry with the factorial $N!$. Directly,

$$1? = 1 = 1!, \qquad 2? = 3 \neq 2!, \qquad 3? = 6 = 3!.$$

Proposition 1 (Same-index termial–factorial equality)**.** *For positive integers, $N? = N!$ only at $N = 1$ and $N = 3$.*

Proof. For $N \geq 1$, division by N gives

$$N? = N! \quad \Longleftrightarrow \quad N + 1 = 2(N-1)!.$$

Direct inspection gives equality for $N = 1$ and $N = 3$, but not $N = 2$. At $N = 4$,

$$2(3)! = 12 > 5.$$

Suppose $2(N-1)! > N + 1$ for some $N \geq 4$. Then

$$2N! = N\,2(N-1)! > N(N+1) > N + 2,$$

which proves the inequality for the next integer. Induction therefore excludes every $N \geq 4$. □

This is a same-index statement. It is distinct from the cross-index question of which factorial values are triangular numbers. The third chamber endpoint is

$$(n, k, m) = (3, 3, 6), \qquad m = 3? = 3!.$$

This finite chamber-boundary coincidence is not a scaling law and is not the cause of the graph-Laplacian observation below.

3 Independent divisibility-graph observation

The divisor graph is an established graph class [2]. Let G_m be the divisibility-comparability graph on $\{1, \dots, m\}$: distinct vertices are adjacent when one divides the other. Let $L(G_m)$ be its combinatorial Laplacian.

Theorem 2. *For $m \geq 4$,*

$$\mathrm{mult}_{L(G_m)}(1) = \pi(m) - \pi(m/2), \qquad \lambda_{\mathrm{gap}} = 1. \tag{2}$$

Proof. Vertex 1 is universal, so G_m is a cone over $H_m = G_m - \{1\}$. The isolated vertices of H_m are exactly the primes p with $m/2 < p \leq m$. Every other vertex lies in one component containing 2: a prime $p \leq m/2$ satisfies $p \sim 2p \sim 2$, while a composite vertex is adjacent to a prime divisor and hence to the same component. Therefore

$$\#\,\mathrm{components}(H_m) = 1 + \pi(m) - \pi(m/2).$$

For a cone, the standard Laplacian join shift [3, 4] turns the zero-eigenspace of $L(H_m)$, after removal of its global constant direction, into the eigenvalue-1 eigenspace of $L(G_m)$. This proves the multiplicity formula. Bertrand's postulate gives $\pi(m) - \pi(m/2) \geq 1$ for $m \geq 4$, so the eigenvalue 1 is present. Every other positive cone eigenvalue exceeds 1, and hence the spectral gap is exactly 1. □

The small spectra are

$$m = 1 : \{0\}, \qquad m = 2 : \{0, 2\}, \qquad m = 3 : \{0, 1, 3\}.$$

The graph object and join-spectrum mechanism are classical. A bounded prior-art scan did not locate this exact packaged multiplicity formula, so it is retained as a hedged proved-here statement rather than a priority claim. The formula is a bounded spectral reformulation of the count of primes in $(m/2, m]$. It does not by itself demonstrate arithmetic structure beyond the evident hub-and-leaf mechanism. A stronger experiment would remove the universal vertex and leaf eigenspace before comparing the residual divisor core with degree-preserving null graphs.

Status note

This supplementary note accompanies Published Working Draft III of the C-Geometry manuscript at version-specific DOI `10.5281/zenodo.21444929`. It is included to preserve independent address-lattice observations without enlarging the novelty boundary of the main geometric paper.

References

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